Mathematics, Information Technologies and Applied Sciences 2014

post-conference proceedings of selected papers extended versions

Editors: Miroslav Hrubý and Šárka Mayerová

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Aims and target group of the conference:
The conference **MITAV 2014** should attract in particular teachers of all types of schools and is devoted to the most recent discoveries in mathematics, informatics, and other sciences as well as to the teaching of these branches at all kinds of schools for any age group, including e-learning and other applications of information technologies in education. The organizers wish to pay attention especially to the education in the areas that are indispensable and highly demanded in contemporary society. The goal of the conference is to create space for the presentation of results achieved in various branches of science and at the same time provide the possibility for meeting and mutual discussions of teachers from different kinds of schools and focus. We also welcome presentations by (diploma and doctoral) students and teachers who are just beginning their careers, as their novel views and approaches are often interesting and stimulating for other participants.

Organizers:
Unity of Czech Mathematicians and Physicists, Brno branch (JČMF),
in co-operation with the Faculty of Military Technology of the University of Defence in Brno,
Faculty of Education of Masaryk University in Brno,
Faculty of Electrical Engineering and Communication of Brno University of Technology,
and Faculty of Economics and Administration of Masaryk University in Brno.

Venue:
Club of the University of Defence in Brno, Šumavská 4, Brno, Czech Republic
June 19 and 20, 2014.

Scientific committee:

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Prof. Karl Hayo Siemsen	Germany
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Programme and organizational committee of the conference:

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Karel Lepka	Masaryk University in Brno, Faculty of Education, Department of Mathematics
Šárka Mayerová	University of Defence in Brno, Faculty of Military Technology, Department of Mathematics and Physics

Conference languages:
Czech, Slovak, English

Programme of the conference:
Thursday, June 19, 2014

12:00-14:00	Registration of the participants
14:00-15:30	Keynote lecture No. 1 (Karl Hayo Siemsen, Germany)
15:30-16:00	Break
16:00-17:30	Keynote lecture No. 2 (Dag Hrubý, Czech Republic)
19:00-22:00	Social event

Friday, June 20, 2014

9:00-14:00	Presentations in sections
14:00	Closing

Each MITAV 2014 participant received printed collection of abstracts **MITAV 2014** with ISBN 978-80-7231-961-9. CD supplement of this printed volume contains all the accepted contributions of the conference.

Now, in autumn 2014, this **post-conference CD** was published, containing extended versions of selected MITAV 2014 contributions. The proceedings are published in English and contain extended versions of 13 selected conference papers. Published articles have been chosen from 47 conference papers and every article was reviewed by two reviewers.

Webpage of the MITAV conference:
http://mitav.unob.cz
http://matika.umat.feec.vutbr.cz/jcmf/?p=683

Content:

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TESTING THE MATHEMATICAL SKILS OF STUDENTS OF ELEMENTARY AND PRE-SCHOOL PEDAGOGY

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Abstract: *Elementary mathematics can cause significant problems to many students of the Pre-school and Elementary Pedagogy, mostly kindergartens and primary schools teachers-to-be. There was a survey realized at the Faculty of Education of Matej Bel University in Banská Bystrica which represents a part of a research within a dissertation thesis intent on preparation of the interactive mathematical study materials. The main goal of the survey was focused on testing how students of Pre-school and Elementary Pedagogy evaluate their own mathematical skills and to determine their real mathematical skills. The study discusses the results of the survey, presents the most frequently occurring mathematical failures of students and compares the self-evaluation of their mathematical skills with their results in mathematical test. The purpose of this study is to present the main weaknesses of elementary mathematical skills of students and scheme out the practical use of the survey's results in the process of developing the interactive mathematical study material for students.*

Key words: Pre-school and Elementary Pedagogy, interactive study material, enhancement of the academic teacher training.

INTRODUCTION

The issue of enhancement of the academic teacher training is currently in the highlight of almost all universities in Slovakia. Faculties of education, particularly those which offer degree programs for Pre-school and Elementary Pedagogy, are trying to improve mathematical preparation of their future teachers. These efforts have to deal with the situation that students who choose this degree program are graduates from various types of secondary schools and not only the number of years allotted to the mathematics study differs but also the quality of mathematical education is very diverse, or it is often quite low. Moreover, sometimes it happens that students miss the Math classes at high schools. This causes serious problems to many university students during their mathematics studies. Very often they are prone to find mathematics unnecessary and think that they will not use it in their practice. Paradoxically, in mathematics needed in practice, students' mathematical skills are deficient. It is obvious that university mathematics cannot substitute neither elementary nor high school mathematics. Nevertheless, the graduation of primary education teachers-to-be from any university while they are still having significant problems in elementary mathematics appears to be a great problem. Therefore, in the winter semester 2013/2014 we began to examine gaps in students' skills in five areas of mathematics primary school education given by the State Curriculum. This survey constitutes a part of the dissertation thesis focused on a design of the interactive mathematics materials which should be used in improving the mathematical preparation of future teachers.

1 STUDENTS AND MATEMATICS

In order to increase the mathematical competence of students and their self-confidence in Mathematics, particularly needed are practical activities, solving both standard and non-standard tasks, tasks analysis and solutions analysis, handling specific objects, etc.., which, with low mandatory lessons of Mathematics during the university study, are not completely comprised. Therefore, universities offer students a number of optional and voluntary courses and the study is often complemented with electronic form, usually through software Moodle.

Students in the Bachelor study are to master a number of topics from arithmetic and geometry. However, many of the students consider these topics unnecessary and useless in their future practice. Hejný after conducted interviews with students notes that secondary and higher Mathematics is perceived as something that must be learnt by heart and after graduating from the faculty, students can forget it all. On the contrary, they understand the importance of Mathematics that is taught at elementary schools. Most of them are convinced that they understand it and they are able to teach it. [5]

Very often these students' beliefs are not consistent with school practice realized during university studies. It often happens that the students are not able to solve tasks intended for the pupils at the first primary education stage and not being able to solve tasks for the second primary education stage is not an exception as well. These facts have led us to the survey of correlation between self-esteem of students and their real skills in particular thematic areas of primary school Mathematics.

2 SURVEY CHARACTERISTICS AND EVALUATION

The purpose of a survey is to identify the main weaknesses of elementary mathematical skills of students of Pre-school and Elementary Pedagogy and to compare the self-confidence of their mathematical skills with their real skills. The survey was conducted by means of two methods – questionnaire and the mathematical test.

2. 1 Questionnaire

To find out the self-assessment of students in area of their mathematical skills, students were asked to fill an electronic questionnaire. The questionnaire was accessed in Slovak language through the Learning Management System *Moodle*. Students were supposed to add one of the following expressions to the questionnaire items: *I am sure I would handle it; I think I could handle it; I think I couldn't handle it; I am sure I wouldn't handle it* and *I cannot say*. Students assessed their skills in all five areas of mathematics primary school education, given by the State Curriculum (1. Numbers, variables and operations with them, 2. Relations functions, tables, diagrams, 3. Geometry, measurement, 4. Combinatory, probability, statistics, 5. Logic, reasoning, proof). Each area was represented by two expressions. So students filled ten questionnaire items concerning the following:

1. To add up two fractions.
2. To determine the price of goods after it was reduced by certain number of percent.
3. To find the relation among given data.
4. To solve the word problem.
5. To use the characteristics of isosceles triangle by solving the problems.
6. To calculate the content area.
7. To list all three-digit numbers from a sequence of digits.

8. To reproach the information from a graph.
9. To create a negation of a statement.
10. To decide about the truthfulness of a statement.

2.1.1 Questionnaire evaluation

The questionnaire was filled out by 44 students of the first year of studies of Pre-school and Elementary Pedagogy. Each answer was evaluated in points. For the answer *I am sure I would handle it*, 4 points, for *I think I could handle it*, 3 points, for *I think I couldn't handle it*, 2 points, for *I am sure I wouldn't handle it*, 1 point and for *I cannot say* 0 points.
The survey showed that students assessed their mathematical skills very optimistically. In the first two items of the questionnaire students received on average more than 3.5 points. It suggests that most students are sure that they would handle the given type of the task or problem. In the items 3. – 6. they received on average 2.6 points. In those tasks the optimism of students decreases. However, a lot of them think that they could handle the tasks. Self-esteem of students increases in the tasks 7. – 10. Average number of points which students in those items received is more than 3. Most students think that they would be able to handle the tasks. Survey's results indicate that students assess their mathematical skills very positively. Average number of received points is 31 out of 40. Only two students reached less than 25 points (both reached 24 points).

2.2 Test

Shortly after students had completed the questionnaire, their actual skills were tested by the written test in Slovak language. The test contained one particular task corresponding to each questionnaire's item. It was taken by 57 students of Pre-school and Elementary Pedagogy.

2.2.1 Test evaluation

1st task: *Calculate* $\frac{2}{3}+\frac{2}{5}$. This task was correctly answered by 89.5% of students. Other 10.5% of students used false methods when fraction's calculating, for example multiplying the numerators and multiplying the denominators or adding numerators and adding denominators. In three tests appeared very surprising results – the whole numbers 2 or 16 – these results indicate that those students have no idea what fractions represent.

2nd task: *The TV costs 400 €. There is a possibility that I can buy it with 30% discount. How much do I spare?* 82.2% of students solved this task correctly and 8.9% incorrectly. The rest of student (8.9%) counted a price of TV after the discount and the answer was absent. By this conclusion we cannot determine if they considered the task completed and considered it to be the spare sum or they did not complete the task.

3rd task: *Put the prices of bananas in relation to their weight into the figure (Fig. 1.)* The figure was correctly filled by 66.7% of students. 19.4% of students properly fill only a part of the figure and did not complete the task. It could be caused due to lack of time or the lack of interest to calculate other values, since there were quite a lot of them. Those students who did not have calculators might felt helpless to count without it. 10.4% of students offered incorrect solutions and two students (3.5%) did not attempt to solve the task.

Fig.1. Figure to the Task 3
Source: own work

Weight (kg)	0,8	1,0	1,3	2,4	2,6	3,1	4,2	4,5	5,0
Price (€)		1,4							

4th task: *If I read 16 pages of a book daily, I will finish the reading in 11 days. How many pages should I read every day if I want to finish it 3 days earlier?* Correct solution was found by 78.9% of students. 17.6% of students solved this task incorrectly. The most students attempted to use the correct algorithm, but they used in inaccurately or in a wrong way. 3.5% of students did not solve this task.

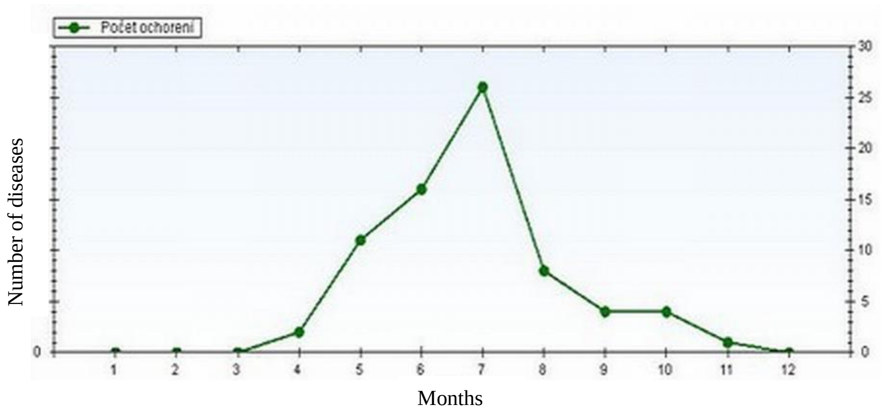
5th task: *In an isosceles triangle, the angle opposite the base has a size of 100°. What are the sizes of other inner angles?* This task solved correctly 66.7% of students. In addition, 12.4% of students calculated a sum of both other inner angels, their answer was 80°. It could have been caused by an inaccurate form of the question. Its form will be changed in the forthcoming survey. The task was solved incorrectly by 19.2% of students and one student did not come up with a solution. Most frequent fails were caused by their conviction that a sum of all inner angles of a triangle is 360°. One student claimed that under given conditions there is no solution because the sum of all inner angels of a triangle is 90°. All students designed a picture but in many cases it was not helpful in their further reasoning. It suggests the fact that a lot of students have no idea about the size of angels.

6th task: *The floor of a bathroom it in a shape of a rectangle with proportions 2.5 m x 3 m. Its height is 2.6 m. How many m^2 of tiles do we need to cover the whole bathroom floor if we would like to cover also both walls (one longer and one shorter) up to the height of 1.5 m?* This task caused serious problems to students. The correct result was found only 17.5% of students. Numerical error in the calculations were made by 10.5% of students and 8.7% of students calculated only area of the floor. 16.1% of students solved the task incorrectly. Most of them used the formulas for the volume, area or some combination of them. 15.5% of students did not even initiate the solution and 31.5% made only a sketch. Such a high percentage of not resolving the tasks may refer to the fact that many students have difficulties when dealing with geometric tasks, respectively they cannot cope with them at all.

7th task: *How many different three-digit numbers can be formed by using following digits: 0,1,2,3?* The task was correctly calculated by only 14% of students. 21% of students begun writing down all the options, but did not succeeded in finding them all. 58% of students calculated the task incorrectly. The most common mistake was including numbers such is 012 into the set of three digit numbers or listing some of the options and considering them for all possible. 7% of students did not make an attempt to solve the task.

8th task: *Based on the information showed in the graph (Pic. 1) determine in what months of the year 2009 more than 10 people got tick-borne encephalitis.* 75.3% of students reproached information from the chart correctly, 17.7% incorrectly and the remaining 7% did not approach the task. The most common mistake was the omission of one month, marking one month in addition or thinking about the half-time of the month.

Occurrence of tick-borne encephalitis
Seasonality.
Year 2009. SR.



Pic. 1. Graph to the Task 8
Source: http://www.epis.sk/AktualnyVyskyt/PrenosneOchorenia/Grafy/Sezonalita.aspx

9th task: *Form the negation of a statement: 'All cars on the park lot are white.'* This task was correctly solved only by 47% of students. The remaining 53% of the students did not find the valid logical negation. On average, erroneous negations were of the following type: *'No car in the parking lot is white.'* Even two students negated the statement by substituting the word *'white'* with *'black'*.

10th task: *The teacher told the students: 'If at least ten of you bring the jumping rope, we will do gymnastics outside the gym class.' Eight out of the children have brought the jumping rope and they did gymnastics the outside gym class. Has the teacher kept her promise from the previous day or not?* This task confused students more than the others tasks. Only 5.3% of them revealed the truthfulness of the implication, 10.4% did not solve the task and 84.2% of students answered the question incorrectly – they support their conviction by the claim that in order to go outside a jump rope should have been brought by ten or more children and only eight did so.

3 COMPARISON OF TEST RESULTS AND SELF-EVALUATION OF STUDENTS

For the assignment of the relation between the self-evaluation and the real level of students' mathematical skills we take into account only those test of the students who filled up the e-questionnaire. The correlation was determined by the Spearman's coefficient of correlation on the significance level of 0.05. We tested the zero hypothesis: *correlation between students' self-evaluation and their test results is zero* on the contrary to the alternative hypothesis: *correlation between the students' self-evaluation and their test results* *is not zero.* Spearman's ρ is 0.125 (Fig. 2) which means, that we cannot refuse the zero hypothesis. The correlation between student's self-evaluation and the real conditions of their mathematical skills is not statistically significant.

Fig.2. Calculation of Spearman ρ
Source: own work

Correlations

			Self-evaluation	Test
Spearman's rho	Self-evaluation	Correlation Coefficient	1,000	,125
		Sig. (2-tailed)		,495
		N	32	32
	Test	Correlation Coefficient	,125	1,000
		Sig. (2-tailed)	,495	
		N	32	32

The survey results are showing that majority of students cannot truly evaluate their mathematical skills. Many students evaluated their mathematical skills as poor but they achieved rather high number of points in the test but there were many of them evaluating themselves as good but their number of points indicates the opposite.

4 USE OF THE SURVEY'S RESULTS

The survey showed that many students have significant limitations in elementary school's mathematics and moreover the majority of them cannot truly evaluate their skills.
The overall percentage of students in tests and their self-assessment is shown the Figure 3.

Fig.3. Student's scores in tests and their self-assessment.
Source: own work

No.	**Test**			**Questionnaire**
	Correctly (%)	**Incorrectly (%)**	**Did not solve (%)**	**Average number of points (max. 4)**
1.	89.5	10.5	0.0	3.7
2.	82.2	17.8	0.0	3.6
3.	66.7	29.8	3.5	2.6
4.	78.9	17.6	3.5	2.6
5.	66.7	31.6	1.7	2.5
6.	17.6	35.3	47.1	2.7
7.	14.0	79.0	7.0	3.5
8.	75.4	17.6	7.0	3.0
9.	47.0	53.0	0.0	3.3
10.	5.3	84.3	10.4	3.1

Students did not perform well in logic, geometry and counting with decimals and the test also revealed significant weaknesses in combinatory. An interesting finding is the fact that 47.1% of students did not solve the task of the surface of a single pattern, the dimensions of which are expressed in decimals. Almost half of all students were discouraged by the task so that only made a sketch or did not solve the task at all.

Marie Tichá claims: *'If we want to change the character of education we will try to operate mainly to teachers, develop and cultivate their professional competence, particularly didactic knowledge of the contents.'* [6] The survey has provided us with many ideas that we will use in the future mathematical preparation of students of Pre-school and Elementary Pedagogy, mostly future teachers. The initial stage of the survey helped us to determine the weakness of the students in the field of Mathematics and directed our attention to the following area:

- provide students with such problems which provide opportunities to acquire the skill of mathematisation of real situations,
- provision of such exercises which enhance critically analyze of reality,
- counting with percentages, fractions, decimals,
- implementation of tasks which develop the combinatorial thinking,
- in the field of logic working with particular statements.

Since the test revealed students' weakness in the given areas, their expansion will be of primary importance (apart from other areas) when developing the interactive mathematical study material for students, which will be used during the course Mathematical Literacy. In this course students of Pre-school and Elementary Pedagogy are expected to develop their mathematical knowledge, skills, mathematical thinking and obtain theoretical foundations and practical experience in the areas of arithmetic and geometry.

5 CONCLUSION

'*To the field study Pre-school and Elementary Education are enrolled mostly students that have some problems with mathematics and its application which are present in the Curriculum for elementary and secondary schools, ...'* [3] This fact confirmed by the survey at the Faculty of Education of Matej Bel University in Banská Bystrica. The survey was launched in the winter semester of the academic year 2013/2014 as a part of the research, which will be a part of the dissertation thesis focused on the creation of the interactive mathematical study materials for students of Pre-school and Elementary Pedagogy. It was realized as a zero phase of design based research and its results will be used in the enhancement of research methods, both questionnaire and test.

The survey showed that many students have significant weaknesses in those areas which are dealt by pupils at primary school. These areas include in particular fractions, percentages, decimals, and combinatory. Some students have difficulties with reading information from a simple graph, confusing the formula to calculate the volume and surface of simple objects or do not have a real idea of the size of angles. Many students cannot even solve logical task. This fact must be taken into consideration in designing interactive materials, because the logic is one of the topics of a course Mathematical Literacy.

The survey also showed that students cannot realistically assess their mathematical skills. Many of them underestimated their mathematical skills and achieved significantly better results than some of those whose self-esteem was markedly higher.

The survey’s results will be used to enhance the academic teacher training of mathematics and to develop the interactive mathematical study material for students, which will be used during the course Mathematical Literacy.

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Stability of the ecosystems models of global processes

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Abstract: We study the stability of the stationary state of the system with delay. The mathematical instruments of investigation we choose the Lyapunovs second method of quadratic functions in the form with the Razumihin's condition.

Keywords: Sufficient condition, asymptotic stability of solution, systems of non-linear differential equations, Markov's process.

Introduction

We propose a mathematical instrument for solution this problem. We consider the non-linear system of differential equations with quadratic non-linearity and the asymptotically stable linear part. One of the first methods in researching the stability of the zero solution of non-linear systems should be considered as methods of linearisation and stability analysis based on the stability of a system of linear approximation. Such types of works were carried out during the second half of the last century, for example, in [1-3]. If the zero solution of the linear approximation is asymptotically stable then in a sufficiently small neighborhood of the equilibrium will be stable and trivial solution of the original non-linear system. In this article we pay attention to systems with a quadratic term. Our results allows us obtain the principle of the serial-greening all spheres of life. Resources of each area are owned and used by its population, and are used to meet the basic material, spiritual, and aesthetic needs, to ensure the health of the population, full of reproduction and the achievement of sustainable development. The principle of funding programs for sustainable development. Stages of implementation of the sustainable development of the territories of any size should be directly related to the release of the secure section of the budgets of areas (countries, regions, cities) separately the costs of protection of the environment in the volume max.

1 Systems without delay

1.1 Statement of problem

Let's consider the following a system with non-linearity of the particular (individual) form, namely systems with quadratic right-hand side, written in vector-matrix form [7-9]

$$\dot{x(t)} = Ax(t) + X^T(t)Bx(t) \tag{1}$$

where $B = \{B_1, B_2, ..., B_n\}^T$ rectangle $n^2 \times n$ matrix consisting symmetric matrix $n \times n$ matrix B_i , $i = 1, 2, ..., n$

$$B_i := \begin{bmatrix} b^i_{11} & b^i_{12} & ... & b^i_{1n} \\ b^i_{21} & b^i_{22} & ... & b^i_{2n} \\ . & . & ... & . \\ b^i_{n1} & b^i_{n2} & ... & b^i_{nn} \end{bmatrix},$$

$X^T = \{X_1(t), X_2(t), ..., X_n(t)\}$- rectangular $n^2 \times n$ - matrix consisting of square $n \times n$ matrix $X_i(t)$, in which the i-x rows are vectors $x(t)$, the other elements are zero,i.e.

$$X_n(t) := \begin{bmatrix} 0 & 0 & ... & 0 \\ 0 & 0 & ... & 0 \\ . & . & ... & \\ x_1(t) & x_2(t) & ... & x_n(t) \end{bmatrix},$$

$$\ddots$$

$$X_1(t) := \begin{bmatrix} x_1(t) & x_2(t) & ... & x_n(t) \\ 0 & 0 & ... & 0 \\ . & . & ... & . \\ 0 & 0 & ... & 0 \end{bmatrix}.$$

Vector and matrix norms define next form

$$\mid x(t) \mid = \left\{ \sum x_i^2(t) \right\}^{\frac{1}{2}},$$

$$\mid B \mid = \left\{ \lambda_{max}(B^T B) \right\}^{\frac{1}{2}},$$

where $\lambda_{max}(\bullet)$ and $\lambda_{min}(\bullet)$ extreme eigenvalues of the corresponding symmetric matrices.
Let the matrix of the linear part of (1) is asymptotically stable. Then, as it follows from the theory of the stability of the linear approximation [10], the zero solution of a non-linear system is also asymptotically stable. If it's taken as a quadratic form $V(x) = x^T Hx$ of the Lyapunov function, then its derivative with respect to the system (1) has the form

$$\frac{dV(x(t))}{dt} = x^T(t) \left[(A^T H + HA) + (B^T X(t)H + HX^T B)\right] x(t).$$

If matrix A is asymptotically stable, then for any positive definite matrix C, there is Lyapunov matrix equation

$$A^T H + HA = -C$$

has a unique solution H a positive definite matrix [10]. Taking in account that H is the solution of the this Lyapunov equation, we obtain

$$\frac{dV(x(t))}{dt} = -x^T(t) \left[C - (B^T X(t)H + HX^T(t)B)\right] x(t),$$

The stability domain of the zero equilibrium is the interior surface of the level of the Lyapunov function $V(x) = r$, $r > 0$ which lies within the area

$$G_0 = \left\{x \in R^n : C - B^T XH - HX^T B > \Theta\right\},$$

where the symbol denotes

$$C - B^T XH - HX^T B > \Theta,$$

is positive definition of matrix.
Let's replace this condition with more "rough". Since, by the selected matrix and vector norms it will be implemented $\mid X(t) \mid = \mid x(t) \mid$ then the total derivative of the Lyapunov function is satisfied

$$\frac{dV(x(t))}{dt} \leq -\left[\lambda_{min}(C) - 2 \mid H \mid\mid B \mid\mid x(t) \mid\right] \mid x(t) \mid^2,$$

Let's denote

$$G_0 = \left\{x \in R^n : \mid x \mid < \frac{\lambda_{min}(C)}{2\lambda_{max}(H) \mid B \mid}\right\},$$

Then the regions of ”guarantee”stability has the form

$$\begin{aligned} G_{r_0} &= max\left\{G_r : G_r \subset G_0\right\}, \\ G_r &= \left\{x \in R^n : x^T H x < r^2\right\}. \end{aligned}$$

If it follows from this dependence, to determine the ”maximum”of stability it should be placed inside a sphere of radius

$$R = \frac{\lambda_{min}(C)}{2 \mid H \mid\mid B \mid}$$

ellipse $x^T H x = r^2$ and ”stretch”$r \longrightarrow \infty$ as long as the ellipse touches the sphere.
We obtain an estimate of the convergence of solutions, the initial position of which is to ”guarantee of stability.”

Theorem 1 *Suppose the matrix of the linear part of system* (1) *asymptotically stable. Then the trivial solution of this system equation is asymptotically stable and for your solutions with initial conditions*

$$|x(0)| < \frac{\gamma(H)}{2|B|\varphi(H)}, \varphi(H) = \frac{\lambda_{max}(H)}{\lambda_{min}(H)}, \gamma(H) = \frac{\lambda_{min}(C)}{\lambda_{max}(H)},$$

are holds next estimates convergence

$$|x(t)| \leq \frac{\gamma(H)\sqrt{\lambda_{min}(H)}|x(0)|}{\left[\gamma(H) - 2 \mid B \mid\mid \varphi(H) \mid |x(0)|\right] e^{0.5\gamma(H)t} + 2 \mid B \mid \varphi(H)|x(0)|}.$$

1.2 Model problem.

Let’s consider the scalar equations

$$\dot{x(t)} = -ax(t) + bx^2(t),$$

with solutions

$$x(t) = \frac{ax(0)e^{-at}}{a - bx(0)\left[1 - e^{-at}\right]}$$

Let’s consider the use of Lyapunov functions with function $V(x) = x^2$. For this function $\lambda_{min}(H) = \lambda_{max}(H) = 1$. full derivative has form

$$\frac{dV(x(t))}{dt} = -2ax^2(t)$$

Estimates convergence (1) for solutions with initial conditions $x(0) < \frac{a}{b}$ has a similar form

$$x(t) \leq \frac{ax(0)}{[a - bx(0)]\, e^{at} + bx(0)} \longrightarrow 0$$

Therefore for this equations exact solutions coincide with estimates given quadratic Lyapunov functions.

1.3 System on the plane.

More positive results of the assessment of convergence of systems with quadratic right-hand side can be obtained by considering a system of the form (1) on the plane. It has the form

$$\begin{aligned} \dot{x_1(t)} &= a_{11}x_2(t) + a_{12}x_2(t) + b^1_{11}x^2_1(t) + 2b^1_{12}x_1x_2 + b^1_{22}x^2_2(t), \\ \dot{x_2(t)} &= a_{21}x_1(t) + a_{22}x_2(t) + b^2_{11}x^2_1(t) + 2b^2_{12}x_1x_2 + b^2_{22}x^2_2(t). \end{aligned}$$

By using notations

$$A := \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad B_1 := \begin{bmatrix} b^1_{11} & b^1_{12} \\ b^1_{21} & b^1_{22} \end{bmatrix}, \quad B_2 := \begin{bmatrix} b^2_{11} & b^2_{12} \\ b^2_{21} & b^2_{22} \end{bmatrix},$$

the last system can be rewritten in the following vector-matrix form

$$\dot{x(t)} = Ax(t) + X^T(t)Bx(t), \tag{2}$$

Then full derivative of the Lyapunov function in the system (2) has the form

$$\frac{dV(x(t))}{dt} \leq -\left[\lambda_{min}(C) - 2 \mid H \mid\mid B \mid\mid x(t) \mid\right] \mid x(t) \mid^2, \tag{3}$$

where

$$\mid H \mid = \lambda_{max}(H) = \frac{1}{2}\left\{h_{11} + h_{22} + \sqrt{(h_{11} - h_{22}) + 4h^2_{12}}\right\},$$

$$\lambda_{min}(C) = \frac{1}{2}\left\{c_{11} + c_{22} + \sqrt{(c_{11} - c_{22}) + 4c^2_{12}}\right\},$$

$$\mid B \mid = \left\{\lambda_{max}(B^T B)\right\}.$$

Guaranteed stability region, as it follows from (3), will be the interior of the ellipse

$$h_{11}x^2 + 2h_{12}xy + h_{22}y^2 \leq r^2_0.$$

1.4 System with a dedicated linear part.

In general, the system (1) with the linear part can be written as [4]

$$\dot{x_i}(t) \ = \ \left[a_i - \sum b_{ij}x_j(t)\right] x_i(t),$$

Here A is the square diagonal matrix with constant coefficients $A = \{a_{ii}\}$, $B = \{B_1, B_2, ..., B_n\}^T$ is the rectangular matrix consisting of symmetric square matrices B_i, in which to place i column is the vector $b_i^T = (b_{i1}, b_{i2}, ..., b_{in})$, $X^T = \{X_1(t), X_2(t), ..., X_n(t)\}$ is rectangular $n \times n^2$ matrix, which consisting of matrix $X_i(t)$, in which the i rows are vectors $x(t)$ other elements zero.
Let's suppose, that $\det B_0 \neq 0$ and

$$B := \begin{bmatrix} b_{11} & b_{21} & ... & b_{n1} \\ b_{12} & b_{22} & ... & b_{n2} \\ . & . & ... & . \\ b_{1n} & b_{2n} & ... & b_{nn}, \end{bmatrix},$$

Then, as a rule, the interest for the searching is a singular point $x_0^T = (x_1^0, x_2^0, ..., x_n^0)$ that is the solution of algebraic equations

$$Bx_0 = a, \quad a^T = (a_1, a_2, ..., a_n)$$

and located in the first quadrant, i.e., $x_i^0 > 0$.
After replacing $x(t) = y(t) + x_0$ and transformation, we get the system of equations of perturbations

$$\dot{y(t)} = \bar{A}y(t) + y(t)^T By(t). \tag{4}$$

Let's suppose that the matrix defined in (4) is asymptotically stable, i.e., $Re\lambda_i(\bar{A}) < 0$. Then the singular point $x_0^T = (x_1^0, x_2^0, ..., x_n^0)$ is asymptotically stable and the region of it's stability can be assessed using a quadratic Lyapunov function $V(y) = y^T Hy$, which is symmetric positive definite matrix is a solution of the Lyapunov equation [5]

$$\bar{A}^T H + H\bar{A} = C.$$

Here C is an arbitrary, symmetric, positive definite matrix.
Taking the total derivative of a function in the system (4), considering that the matrix is a solution of the Lyapunov equation, we obtain

$$\frac{dV(y(t))}{dt} \leq -\left[\lambda_{min}(C) - 2 \mid H \mid\mid B \mid\mid y(t) \mid\right] \mid y(t) \mid^2,$$

And, in the case of asymptotic stability of the matrix $\bar{A}$, the guaranteed stability of the equilibrium area of the singular point is inside the ellipse $y^T Hy = r^2$ is inside the sphere $\mid y \mid = R$. Denoting

$$G_0 = \left\{ y \in R^n : \mid y \mid < \frac{\lambda_{min}(C)}{2 \mid H \mid\mid B \mid} \right\},$$

we find that the area "guarantee"stability has the form [4]

$$G_{r_0} = max \left\{ G_r : G_r \subset G_0 \right\},$$
$$G_r = \left\{ y \in R^n : x^T Hx < r^2 \right\}.$$

As it follows from the relation (1.4), ellipse $y^T Hy = r^2$ should be placed inside a sphere of radius

$$R = \frac{\lambda_{min}(C)}{2 \mid H \mid\mid B \mid}$$

and "stretched"$r \longrightarrow \infty$ as long as the ellipse touches the sphere.
We obtain estimates convergence of solution initial state lay in 'guaranty' stability regions.

2 Systems with delay

Typically, the models of the economy and the environment inherent lag factor, defined "the time of puberty,"or "the time of the decision". Are therefore more appropriate mathematical model describing the system of functional differential equations with delay [6-7]. One of the first mathematical models described by differential equations with constant delay, were the equation Hutchison and Voltaire [1-3].

2.1 Verhulst equation with delay.

Verhulst equation displays the dynamics of population growth with saturation. Limited growth due to "internal competition."Note the following factor, specifying the model Verhulst. Competition generally occurs between the new population and the population born with retardation. In this case, population dynamics is determined by the equation Hutchison (1948), which has the form of a differential equation with delay

$$\frac{dx(t)}{dt} = ax(t)\left(1 - \frac{x(t-\tau)}{k}\right). \tag{5}$$

The delay is due to the finite time required to achieve the "time of puberty."
Dynamical system described by equation (5) has two equilibrium $x(t) \equiv 0$ and $x(t) \equiv k$. It is easy to see that the linear approximation is given by the equation at the point $x = 0$ and points to the instability of the zero equilibrium. Consider the second point of rest $x(t) \equiv k$.
Draw linearisation

$$\frac{dx(t)}{dt} = f(x(t), x(t-\tau)),$$

in the neighbourhood $x(t) \equiv k$.
After transformation and substituting the corresponding values we get

$$\frac{dx(t)}{dt} = -a\left[x(t-\tau) - k\right].$$

And in the neighbourhood of a singular point $x(t) = k$ of the equation of the linear approximation of the form

$$\frac{dy(t)}{dt} = -a\left[y(t-\tau) - k\right].$$

where $y(t) = x(t) - k$. The characteristic equation is

$$\lambda + ae^{\lambda\tau} = 0$$

And, as a consequence of [6,7], with

$$0 < a\tau < \frac{\pi}{2}$$

equilibrium $x(t) = k$ is locally asymptotically stable.
Estimate the stability region in the phase space of the equilibrium position $x(t) = k$ of the original non-linear system (5). After the transformation $x(t) = y(t) + k$ point $x(t) = k$ to the origin, we obtain the equation

$$\frac{dy(t)}{dt} = -a\left[y(t) + k\right]y(t-\tau).$$

We use a quadratic Lyapunov function $V(y) = \frac{1}{2}y^2$. Since we consider the delay equation, the total derivative of the evaluation will be used Razumihin's condition [4]. This condition means geometrically that the total derivative is calculated, subject approach solutions from the inside surface of the level of the Lyapunov function. For the function $V(y) = \frac{1}{2}y^2$ it has the form

$$| \, y(t-\tau) \, | < | \, y(t) \, | \, .$$

And the total derivative of the Lyapunov function along a solution has the form

$$\frac{dV(x(t))}{dt} \leq -a \left[1 - \frac{1}{k} \mid y(t) \mid \right] \mid y(t) \mid^2 + \frac{a}{k} \mid y(t) \mid [\mid y(t) \mid + k] \mid y(t) - y(t-\tau) \mid .$$

We estimate the phase coordinates with delay and without. Rewrite previously inequality in next form

$$\frac{dV(x(t))}{dt} \leq -a \left[1 - \frac{1}{k} \mid y(t) \mid \right] \mid y(t) \mid^2 + \frac{a}{k} \mid y(t) \mid [\mid y(t) \mid + k] \mid y(t) - y(t-\tau) \mid .$$

Thus, when

$$a \left[1 - \frac{1}{k} \mid y(t) \mid \right] > \left(\frac{a}{k} \right)^2 [\mid y(t) \mid]^2 \, \tau$$

the total derivative of the Lyapunov function is negative definite. Thus, the stability conditions are determined by the inequalities

$$| \, y(t) \, | < k,$$

$$\tau < \frac{k[k - \mid y(t) \mid]}{a[\mid y(t) \mid + k]^2}.$$

2.2 General quadratic model with delay.

In the universal vector-matrix form, the quadratic model with delay is written as

$$\dot{x(t)} = \left[Ax(t) + x^T(t-\tau) \right] Bx(t),$$

Suppose, as in the case without delay, $x_0^T = (x_1^0, x_2^0, ..., x_n^0)$ is the solution of algebraic equations

$$B_0 x = a, \quad a^T = (a_1, a_2, ..., a_n)^T$$

Making the substitution in $x(t) = y(t) + k$, we obtain the system of equations of the perturbation which, after transformation, takes the form

$$\dot{y(t)} = \bar{A}y(t) + y^T(t-\tau)By(t). \tag{6}$$

$$\bar{A} := \begin{bmatrix} b_{11}x_1^0 & b_{21}x_1^0 & ... & b_{2n}x_1^0 \\ b_{21}x_2^0 & b_{22}x_2^0 & ... & b_{2n}x_2^0 \\ . & . & ... & . \\ b_{1n}x_n^0 & b_{1n}x_n^0 & ... & b_{nn}x_n^0 \end{bmatrix}.$$

We study the stability of the zero equilibrium state of the system (6) using the method of Lyapunov functions quadratic form $V(y) = y^T Hy$. In assessing the total derivative is used Razumihin's condition [8]. For the function $V(y)$ it has the form

$$\mid y(t-\tau) \mid \leq \sqrt{\varphi(H)} \mid y(t) \mid .$$

The set of points $y \in R^n$ that are within the level surfaces $V(y) = \alpha$ of a Lyapunov functions $V(y) = y^T Hy$ by V^α , and its boundary by ∂V^α , i.e.

$$V^\alpha = \{y \in R^n : V(y) < \alpha\}$$

Theorem 2 *Let the solutions $y(t)$ of (6) is performed $y(T) \in \partial V^\alpha$ at the time $t = T > \tau$,and at $-\tau \leq t < T$ will $y(t) \in V^\propto$. Then holds the inequality*

$$|y(T) - y(T-\tau)| \leq \left[\bar{A} + B\sqrt{\varphi(H)}y(T)\right] \sqrt{\varphi(H)}y(T)\tau.$$

Theorem 3 *Let the $-\tau \leq t \leq 0$ initial conditions $\varphi(t)$ for the solutions $y(t)$ is implemented $\varphi(t) < \delta$. Then this solutions $y(t)$ on interval $0 \leq t \leq \tau$ be implemented*

$$|y(t)| \leq \delta \exp\left[\bar{A} + B\delta\right]\tau.$$

Theorem 4 *Let the matrix $\bar{A}$ is asymptotic stability: $Re\lambda_i(\bar{A}) < 0$. Then as $\tau < \tau_0$, where*

$$\tau_0 = \frac{\lambda_{min}(C)}{2HB\sqrt{\varphi(H)}},$$

equilibrium is asymptotic stable.

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RAISING ATTRACTIVENESS OF STUDIES OF NATURAL SCIENCE SUBJECTS BY UTILIZATION OF COMPUTING SUPPORT

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Abstract: *In the present time pupils of primary schools as well as students of secondary schools and universities show their disinterest, even aversion to the studies of natural sciences and technical subjects such as mathematics, physics, mechanics, elasticity and strength and so on. One of the possibilities how to make teaching these subjects more attractive and efficient is introduction of new teaching methods into educational process. That is how these subjects can become more interesting, imaginable and understandable for pupils and students. The paper deals with the possibility of introduction and utilization of modern information technologies into educational process, where the possibility of solving problem in physics (the part of the mechanics of dynamic systems) is shown by using program tools (e.g. MATLAB, Mathematica, MS Excel).*

Keywords: modern information technologies, MATLAB, numerical solution of problem

INTRODUCTION

Mathematical and physical basis is a necessary condition for successful management of modern technical disciplines. In general disinterest in mathematics and physics along with the extent reduction of teaching these subjects at primary and secondary schools is proved by worse preparedness of secondary school graduates for the studies of natural sciences and technical disciplines and lower motivation to study technical study programs at technical universities. One of the forms to increase the interest in studies of natural sciences subjects is also application and utilization of innovative teaching forms, mainly introduction and utilization of information and communication and software tools into teaching. Information and communication technologies provide incomparably bigger information basics as it was several years ago. This gradually changes the style of teaching and makes teachers implement new technologies not only in real pedagogical action, but also at its preparation. By convenient combination of traditional and modern teaching methods we can stimulate interest of students in natural sciences studies, create conditions for the individualization of education and improve conditions for increasing the quality of education. Development of computer technique introduces many possibilities of utilization of software support at educational process.

1. INFORMATION TECHNOLOGIES IN EDUCATION

At the present time the topic of computers utilization in educational process is discussed very often at various levels. If we should summarize the results of those discussions, it could be said the computers:

- form reliable and attractive environment for learning
- provide positive feedback

- help to create shapely correct text
- respect individual requirements, pace, speed and skills
- allow to go back to a problem and start or complete work at various places
- help in learning of students with specific disorders of learning and disabled students
- make rich information sources accessible
- comprehensibly present complex thought processes and relations through graphics
- offer environment for development of students thinking

According to I. Turek [1] computer has a stable place in present-day school as:

1. working tool – with its aid students learn a lot of skills needed for their progress (knowledge and skills in using text, graphic, tabular editors, various application programs)
2. communicative tool – enables students to communicate at various distances through Internet and e-mail
3. teaching aid – it is possible to classify it in various stages of teaching process
   - as motivational factor
   - as means at acquisition of a new subject matter
   - as means at revision and consolidation of a subject matter
   - as test and evaluation means
   - as simulator of problem situations
4. helping tool at working out some psychical problems of students – a lot of students work with computer in more relaxed way, without stress from their own failure and more openly communicate with teacher
5. tool supporting creativity – in various parts of students activities
6. information source from various areas of life

Information technologies caused fundamental changes in many areas of human activity. As the graduates of a modern school should know to use IT in their future occupation, changes had to take place in educational area. Information technologies apply not only to students, but also teachers. Teacher obtained tools and new possibilities for his work [2]:

- better preparation for teaching (information can be drawn from multimedia CDs)
- teaching the subject with the utilization of PC (animations, educational programs, tests, micro labs)
- participation in teleprojects and information gathering through Internet (discussion clubs, information services on www)
- presentation of one´s own results on web sites.

1.1 Utilization of information technologies in teaching physics

Because of above mentioned features it is suitable to use IT also in teaching physics. If personal computers are supplemented with suitable program accessories or extended peripheral accessories, they can function as [3]:

- **Means of communication**
  Personal computer connected into the net of other computers presents quick and effective means of communication. It provides its user with services of active communication with other users: e-mail, telephony. Information can also be exchanged by conference, videoconference or chat participants. Passive services are of the same importance, e.g. information gathering through web sites services.
- **Tool for data acquisition and elaboration**
  An experimental method is one of the basic methods of knowledge in physics. It is the same in school physics, when a student discovers unknown or verifies learnt knowledge,

phenomena and laws. From this standpoint the question of physical measurements is of great importance. Through those measurements we obtain physical data, which are necessary to be elaborated and interpreted in a suitable way. By introducing computers in schools, teaching physics gains an effective tool for data acquisition and elaboration. Programs, which were created for various utilization in practice, such as tabular processors, statistical and computer programs, can be used for data elaboration. By their use students can elaborate big data files and also accord mathematical knowledge with physics needs. If a school has additionally suitable technical equipment, computer is able to scan data from a real physical experiment, elaborate them quickly and display them graphically. The basic technical means, by which computer is needed to be equipped, are the following ones [1]:

– interface card, by which a user obtains widely usable laboratory apparatus for data acquisition and control of experiments
– set of sensors, which transform various physical quantities on voltage, as computer is able to identify only the level of voltage. For example location, velocity, force, pressure, temperature can be measured
– program environment, which accords individual activities as data scanning and elaboration, or physical phenomena modeling, enables to display measured data at the measurement process in tables, graphs, elaborate them further, analyze measured dependences, differentiate, integrate, filter and so on (Dutch IP COACH or Czech ISES).

- **Presentation aid**

  Presentation comprises a series of photographs with text, pictures, tables, animations, video, sounds and so on. The photographs are gradually displayed through the monitor of computer, television circuit or data projector. Various timing of objects illustration, effects of animations, starting possibilities can be set on every photograph. Presentation can be used as a guide to explanation of a subject matter. The parts of a subject matter, in which classical data record on the blackboard is used, are prepared by presentation, where individual patterns, definitions, assignments of tasks or pictures are displayed through clicking a mouse. Teacher can pay more attention to oral commentary on given facts and work with a class. Moreover, the text replenished with pictures, animations, schemes, photographs is more interesting and illustrative for a student. Presentation can replace time-consuming setting of problems, numerical problems and so on, when students require to repeat setting of values and wording of questions. Presentation does not have to be used only by a teacher, but also by students who can prepare the papers for lessons, end-of-year works and present them in more interesting way as only by reading from the paper.

- **Carrier of educational environment**

  Many programs serve for teaching physics and their use is very broad. There are multimedia programs elaborated in encyclopedic way. Their use at a teaching lesson is indeed limited, but they cover the subject matter of physics abundantly. Multimedia programs also provide many ideas how to bring school physics closer to everyday life of students. They can be used by teacher in his preparation for teaching or by students at their individual work (preparation of papers, project education and so on).

  Furthermore, there are programs dealing with computer modeling and simulation of physical phenomena. Students have two-dimensional laboratory with various objects and can change the conditions, they are in (e.g. value of gravity acceleration, air resistance). In the process of animated action it is possible to follow the values of individual quantities in graphs.

  There are many programs, in which student moves, receives information, solves various problems, answers to questions and is also evaluated. It is ideal at such programs, when student sits in front of computer alone. If more students work with the same program at the same computer, efficiency of education disappears.

2. MATLAB PROGRAM TOOL

Efficiency of educational process can be increased by application of some modern teaching methods. One of them is implementation of information and communication technologies into teaching such as utilization of means of MATLAB at solving differential equations.
MATLAB presents highly efficient language for technical calculations. It combines calculations, visualisation and programming into simply usable environment. It is an interactive tool in which the basic data type is the field without necessity to declare its parameters. This property together with number of in-built functions enables relatively easy solution of many technical problems. In school environment MATLAB is a standard tool in teaching mathematics and other technical subjects, but it is also an efficient tool for research, development and data analysis [4], [5], [7], [8].
MATLAB is closer to the programming language compared to other similar products. From didactic point of view it is a suitable system, because it does not require complicated programming formulae and after relatively short time a beginner can manage to work in MATLAB. On the other side it presents a strong tool for experienced users. MATLAB does not have so many prearranged mathematical functions as for example MATHEMATICA. It does not have integrated properties such as MathCad. Support of symbolic calculations is not its standard part as it is at above mentioned products. However, It does not mean that MATLAB is depleted of these possibilities. It contains more than 500 simple or more complex mathematical functions implemented in the form of highly efficient and robust algorithms. From these functions it is possible to compose arbitrarily other functions. Sets of functions suitable for solution of a certain type of problems in MATLAB are called toolboxes. SIMULINK is an independent extension of MATLAB – solution of the system of nonlinear differential equations with a graphic entry of the system being solved. It enables graphically to observe dependencies of parameters at any connection point. It is used for simulation of dynamic behaviour of the observed system. It is possible to use MATLAB in case of robust calculations, processing of extensive data files, work with large matrices and in cases when solution of the problem can be converted into vector and matrix operations. With regard to programming possibilities it is advantageous to use MATLAB also in case of branched or iterative algorithms of solution.

3. PHYSICAL ANALYSIS OF THE PROBLEM

Problem: An object is thrown out of the plane flying in h height at v velocity. What is the falling distance of the object? Examine the trajectory of motion of the thrown object.

Solution: Motion of the object thrown out of the plane in a certain height above earth´s surface is considered. Solution of the problem arises from the fact that the object performs curvilinear motion in the gravitational field of the Earth that is called horizontal throw (Fig. 1).

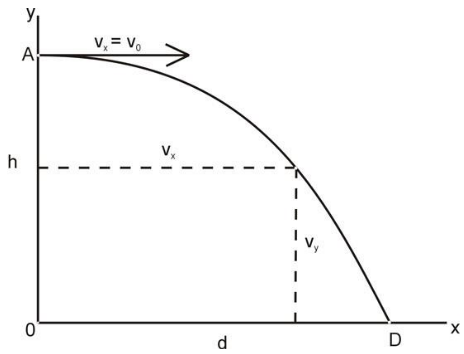

Fig. 1. Horizontal throw

It is the motion composed of two motions: uniform rectilinear motion in horizontal direction and vertical throw downwards with v_0 initial velocity. Trajectory is a part of the parabola with the peak at the place of throw [6].
Balance of forces is the basis to write the equations describing trajectory of the object. For balance of forces in the direction of the x axis we have

$$F_x + F_{tx} = 0 \tag{1}$$

where F_x is under Newton´s second law the acceleration force in the direction of the x axis, F_{tx} is the resistive force in the direction of the x axis.
For balance of forces in the direction of the y axis we have

$$F_y + F_{ty} + F_g = 0 \tag{2}$$

where F_y is under Newton´s second law the acceleration force in the direction of the y axis, F_g is the gravitational force, for which size we have $F_g = mg$ and F_{ty} is the resistive force in the direction of the y axis.
For the size of the resistive force under Newton´s law we have

$$F_t = \frac{1}{2} c \rho S v^2 \tag{3}$$

where c is the coefficient of resistance which depends on the shape of the object, ρ is the density of environment (air), S is the effective cross-section of the object (in case of the object of circular section $S = \pi r^2$) and v is velocity.

For the model given based on the equations (1) and (2) we get the system of nonlinear differential equations of second order:

$$\begin{aligned} m\frac{d^2x}{dt^2} &= -F_t \\ m\frac{d^2y}{dt^2} &= -F_t - mg \end{aligned} \tag{4}$$

Analytic solution of the system of differential equations is difficult, it requires considerable mathematical knowledge and skills from the theory of solving differential equations and obviously does not lead to simple dependencies and results. Therefore the problem will be solved numerically by using MATLAB [4], [9], [10].

4. SOLUTION OF THE PROBLEM IN MATLAB

To solve the problem in MATLAB, it is necessary to convert the system of two differential equations of second order into the system of four differential equations of first order

$$\frac{dx}{dt} = v_x \qquad \frac{dy}{dt} = v_y$$
$$\frac{dv_x}{dt} = -\frac{F_t}{m}\frac{v_x}{\sqrt{v_x^2 + v_y^2}} \qquad \frac{dv_y}{dt} = -\frac{F_t}{m}\frac{v_x}{\sqrt{v_x^2 + v_y^2}} - g \tag{5}$$

At solving the problem the system of equations was written into MATLAB and trajectory of motion of the falling object was examined.
The first step of the problem solution is entry of initial parameters of the problem and creating function that determines derivation of the function in time. The script was created, in which initial conditions for solving the system of differential equation were given. To solve the problem *ode45* function was used. It is a basic standard function for solving differential equations and its syntax is

$$[t, y] = ode45('function\_name, time\_interval, initial\_conditions)$$

where *function_name* refers to the function describing the system of differential equations, parameter of *time_interval* presents the vector with two elements – t_0 initial time of solution and t final time of solution, parameter of *initial_conditions* presents y_0 vector of initial conditions so that we have $y(t_0) = y_0$.

Two parameters are the output *ode45* function:
t - vector, which contains time intervals, in which values of solution are determined
y – matrix, which contains its own solutions, where the number of lines of *y* matrix corresponds with the number of lines of *t*, the number of columns corresponds with the number of equations of the system being solved.

The following initial parameters are used at the problem solution: object weight $m = 10kg$, air density $\rho = 1{,}276 kgm^{-3}$, resistance coefficient $c = 0{,}17$, height above earth´s surface $h = 2km$, velocity $v = 60ms^{-1}$, object radius $r = 10cm$, projectile section $S = \pi r^2$.

```
options =  odeset('Events', @events);
[t, sv] = ode45(@dif_rce, [0, t_end], [sx0, sy0, vx0, vy0], options);
sx = sv(:,1);
sy = sv(:,2);
vx = sv(:,3);
vy = sv(:,4);
fig1 = figure;
plot(sx, sy, 'b');
grid on;
title('Trajectory of motion');
xlabel('s_x (m)'); ylabel('s_y (m)');
fprintf('\nFalling distance of the object sx = %4.0f m\n', max(sx));
```

The result of launching the program in MATLAB is the printout of numerical value of falling distance of the object and drawing the graph of trajectory of the falling object (Fig. 2).

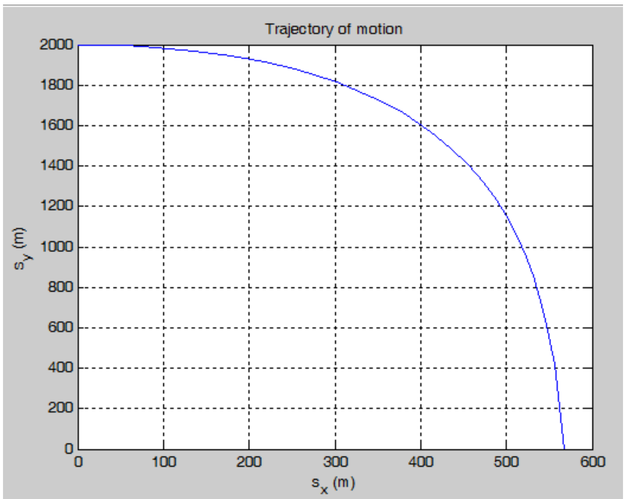

Fig. 2. Trajectory of motion of the object

The printout of calculated numerical value of falling distance of the object:

>> Falling distance of the object s_x = 567 m

The script was created to draw dependence of trajectory and velocity of the falling object on time.

```
fig1 = figure;
subplot(1, 2, 1);
plot(t, sx, 'b', t, sy, 'r');
grid on;
title('Dependence of trajectory on time');
xlabel('t (s)'); ylabel('s (m)');
legend('s_x', 's_y');

subplot(1, 2, 2);
plot(t, vx, 'b', t, vy, 'r');
grid on;
title('Dependence of velocity on time');
xlabel('t (s)'); ylabel('v (m/s)');
legend('v_x', 'v_y');
```

After launching the script to draw graphs of dependencies of trajectory and velocity on time we get the following dependencies (Fig. 3):

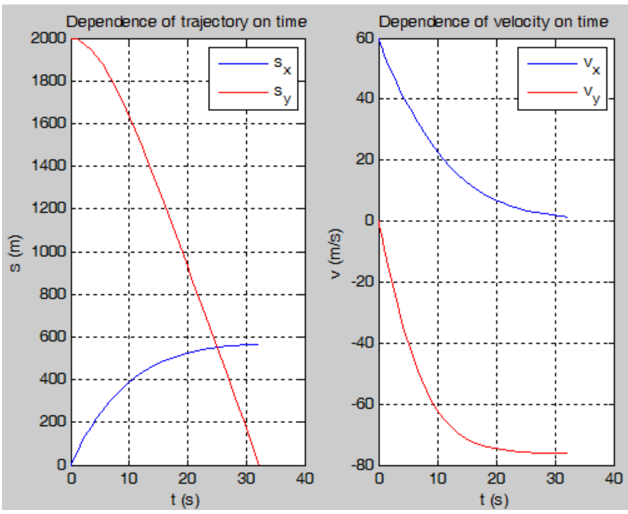

Fig. 3. Dependence of trajectory and velocity on time

CONCLUSION

Computers are modern communication media, which influence present-day young people, and therefore they are an excellent motivational factor for pupils and students. It is suitable to use computers to reach one´s goals in teaching physics, but consciously and coherently. Computer cannot replace a teacher, who guides students, draws their attention to the main point, systemizes knowledge of students, offers students opinions and recommends methods for individual work, but teaching lesson with unconventional form of work is immensely interesting and motivating for pupils and students. Presence of information technologies in educational process has a positive influence on efficiency of educational process. Students accept them and evaluate them very positively.

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APPLICATION OF STATISTIC METHODS AT ANALYSIS OF THE PROCESS OF ALUMINIUM NICKEL ELECTROPLATING

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Abstract: *The paper deals with the application of cluster analysis at evaluating the influence of technological conditions of the process of aluminium nickel electroplating EN-AW 1050 A H24 to the quality of formed layer. For this purpose methods of evaluating the entire quality of formed coating was proposed, which creates conditions to set up a suitable data matrix for application of cluster analysis as well as other statistic methods that enable to analyze and predict the entire quality of eliminated coating in relation to technological conditions of its formation.*

Keywords: cluster analysis, design of experiments, nickel electroplating, aluminium

INTRODUCTION

Nickel electroplating is a commercially important and versatile surface-finishing process. Its commercial importance may be judged from the amount of nickel in the form of metal and salts consumed annually for electroplating, now roughly 100,000 metric tons worldwide, as well as its versatility from its many current applications [1]. The applications of nickel electroplating fall into three main categories: decorative, functional, and electroforming. In decorative applications, electroplated nickel is most often applied in combination with electrodeposited chromium. The thin layer of chromium was first specified to prevent the nickel from tarnishing. It was originally deposited on top of a relatively thick, single layer of nickel that had been polished and buffed to a mirror-bright finish. Today decorative nickel coatings are mirror bright as deposited and do not require polishing prior to chromium plating. Multilayered nickel coatings outperform single-layer ones of equal thickness and are widely specified to protect materials exposed to severely corrosive conditions. The corrosion performance of decorative, electroplated nickel plus chromium coatings have been further improved by the development of processes by which the porosity of chromium can be varied and controlled on a microscopic scale (microdiscontinuous chromium). Modern multilayered nickel coatings in combination with microdiscontinuous chromium are capable of protecting steel, zinc, copper, aluminium, and many other materials from corrosion for extended periods of time [2], [3]. The complexity of modern-day nickel plus chromium coatings is more than offset by the greatly improved corrosion resistance [4] that has been achieved without significantly increasing coating thickness and costs. There are many functional applications where decoration is not the issue. Instead, nickel and nickel alloys with matte or dull finishes are deposited on surfaces to improve corrosion and wear resistance or modify magnetic and other properties. The properties of nickel electrodeposits produced under different conditions of operation are of particular interest in this connection. Electroforming is electroplating applied to the fabrication of products of various kinds. Nickel is deposited onto a mandrel and then removed from it to create a part made entirely of nickel. A variation of this is electrofabrication where the deposit is not separated from the substrate and where fabrication may involve electrodeposition through masks rather than the use of traditional mandrels. The many current applications of nickel electroplating are the result of developments and improvements that have been made almost since the day the process was discovered.

Bottger developed the first practical formulation for nickel plating, an aqueous solution of nickel and ammonium sulphates in 1843, but earlier references to nickel plating can be found. Bird apparently deposited nickel on a platinum electrode in 1837 from a solution of nickel chloride or sulphate, and Shore patented a nickel nitrate solution in 1840 [1]. The solution developed by Bottger remained in commercial use for 70 years, however, and he is acknowledged to be the originator of nickel plating [1]. Dr. Isaac Adams, Jr., a medical doctor educated at Harvard University and at the L´École de Médicine in Paris, was one of the first to commercialize nickel plating in the United States, and his patented process gave his company a virtual monopoly in commercial nickel plating from 1869 to 1886. His patent covered the use of pure nickel ammonium sulphate. Although Adams's solution was similar to Bottger's, his emphasis on operating the bath at neutral pH was undoubtedly vital for controlling the quality of the nickel deposited, since excessive amounts of ammonia would tend to lower cathode efficiency and embrittle the deposit. Largely as a result of the publicity generated by Adams, nickel plating became known worldwide, and by 1886, the annual consumption of nickel for plating had grown to about 135 metric tons [8], [9]. Remington, an American residing in Boston, attempted to market a nickel ammonium chloride electroplating solution in 1868, but perhaps of greater significance, in view of subsequent developments, were his attempts to use small pieces of electrolytic nickel as an anode material in a platinum anode basket [1]. Weston introduced the use of boric acid and Bancroft was one of the first to realize that chlorides were essential to ensure efficient dissolution of nickel anode materials [1]. Professor Oliver P. Watts at the University of Wisconsin, aware of most of these developments, formulated an electrolyte in 1916 that combined nickel sulphate, nickel chloride, and boric acid and optimized the composition of the nickel electroplating solution [1]. The advantages of his hot, high-speed formula became recognized and eventually led to the elimination of nickel ammonium sulphate and other proprietary solutions. Today the Watts solution is widely applied, and its impact on the development of modern nickel electroplating technology cannot be overstated. Decorative nickel plating solutions are variations of the original Watts formulation, the main difference being the presence in solution of organic and certain metallic compounds to brighten and level the nickel deposit. Because of his use of organic additives like benzene and naphthalene di- and trisulfonic acids, Schlotter's decision to market a bright nickel plating solution in 1934 is a milestone in the commercial development of decorative nickel plating [1].

1. CONDITIONS OF EXPERIMENTAL VERIFICATION

Experimental verification of methods of quality evaluation of eliminated coating was carried out in the Hull Cell (Fig. 1). The Hull Cell is an extremely powerful test tool capable of controlling several plating variables at the same time and it has probably been the tool that has contributed in more extent to the development of electroplating. This Cell is a trapezoidal box of non-conducting material that in its standard size holds 267 ml of solution. The anode is laid against the right angle and the cathode is laid against the sloping side.

Thus, when a current is passed through the solution, the current along the sloping cathode varies in a known way. The cell shows the limits of acceptable current density ranges, detects the presence of impurities, degree of levelling or ability of electroplating process to deposit smooth uniform coating on a rough surface and throwing power or uniformity of the thickness of a coating deposited on irregularly shaped part. It also controls the morphology of the deposit, the alloy composition as a function of current density, agitation effects or the cathode average efficiency, that is, the ratio of the actual amount of the deposited material to the theoretical amount that should be deposited. It also evaluates the covering power of competitive electroplating systems, that is, the ability of an electroplating bath to produce a coating at a low current density electroplating range, the heat stability, life, compatibility, etc. and even it allows to make an estimation of the concentration of the compounds involved in electrodeposition.

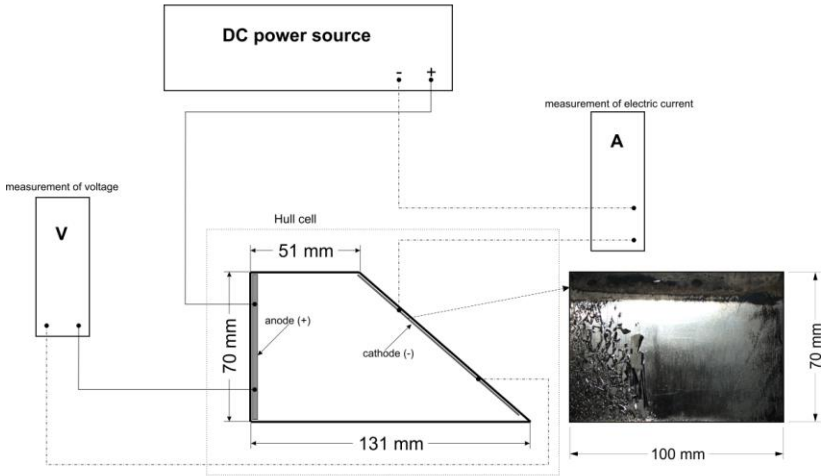

Fig. 1 Hull Cell and experimental equipment
Source: own

Experiment No.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
1	55	21	3	1	3	70	50	2
2	155	7	11	5	3	70	50	5
3	55	7	3	1	3	40	10	2
4	55	21	11	5	3	40	10	5
5	55	21	3	5	1	70	50	2
6	155	7	11	1	1	40	50	2
7	155	21	3	5	3	40	50	5
8	155	21	11	1	1	40	10	2
9	155	7	3	5	1	40	10	2
10	55	7	3	1	1	70	10	5
11	55	21	11	1	3	40	10	5
12	155	7	3	1	3	40	50	5
13	55	7	3	5	1	40	50	5
14	155	21	3	5	3	70	10	2
15	155	21	3	1	1	70	10	5
16	55	7	11	1	3	70	50	2
17	155	7	11	5	3	70	10	2
18	55	7	11	5	1	70	10	5
19	155	21	11	1	1	70	50	5
20	55	21	11	5	1	40	50	2
21	105	14	7	3	2	55	30	3,5
22	105	14	7	3	2	55	30	3,5

Tab. 1 Experiment Plan (x_1 - $m(NiCl_2)$ – amount of nickel dichloride in $g.l^{-1}$, x_2 - $m(H_3BO_3)$ – amount of hypoboric acid in $g.l^{-1}$, x_3 – basic glitter additive in $ml.l^{-1}$, x_4 – levelling glitter additive in $ml.l^{-1}$, x_5 – wetting agent in $ml.l^{-1}$, x_6 – electrolyte temperature in °C, x_7 – electroplating time in min, x_8 – voltage in volts)
Source: own

Experimental verification was carried out at the process of electrolytic nickel plating on aluminium samples. Chloride electrolyte based on hypoboric acid was used. The experiment was carried out in accordance with DoE (Design of Experiment) methods, at which full factor experiment based on reduced replicas with 22 individual experiments was selected (Tab. 1). Last 2 experiments presented experiments in the "middle" of experimental environment with setting up individual factors on "zero" level.

EN-AW 1050 A H24 aluminium was used as a material of the cathode (sample). Chemical composition of the experimental material can be found in Tab.2.

Si	**Fe**	**Cu**	**Mn**	**Cr**	**Zn**	**Ti**
0,25	0,40	0,05	0,01	0,01	0,07	0,05

Tab. 2 Chemical composition of the experimental material in wt. %

The samples with dimensions 100×70×0,5 mm were chemically degreased in solution containing: sodium bicarbonate < 20 %, sodium metasilicate pentahydrate < 5 %, phosphates < 30 %, borates < 40 % and surfactants, at the temperature 50 ± 2 °C for total exposure time 15 minutes. The samples were then rinsed thoroughly in deionized water and immersed in a 45 % sodium hydroxide solution at a temperature 55 ± 2 °C for 1 min.
The layer of zinc in the solution formed by oxide zinc (75 $g.l^{-1}$) and sodium hydroxide (400 $g.l^{-1}$) was created on all samples before the process of nickel electroplating. Individual samples spent constant time of 2 minutes in the solution.

2. METHODS OF EVALUATING COATING QUALITY

The standard size of the sample designed for electroplating in the Hull Cell is the same for all experiments and it is 100×70×0,5 mm. Electroplated sample is schematically shown in Fig. 2, where basic characteristics used at deducing methods of evaluating surface are defined.

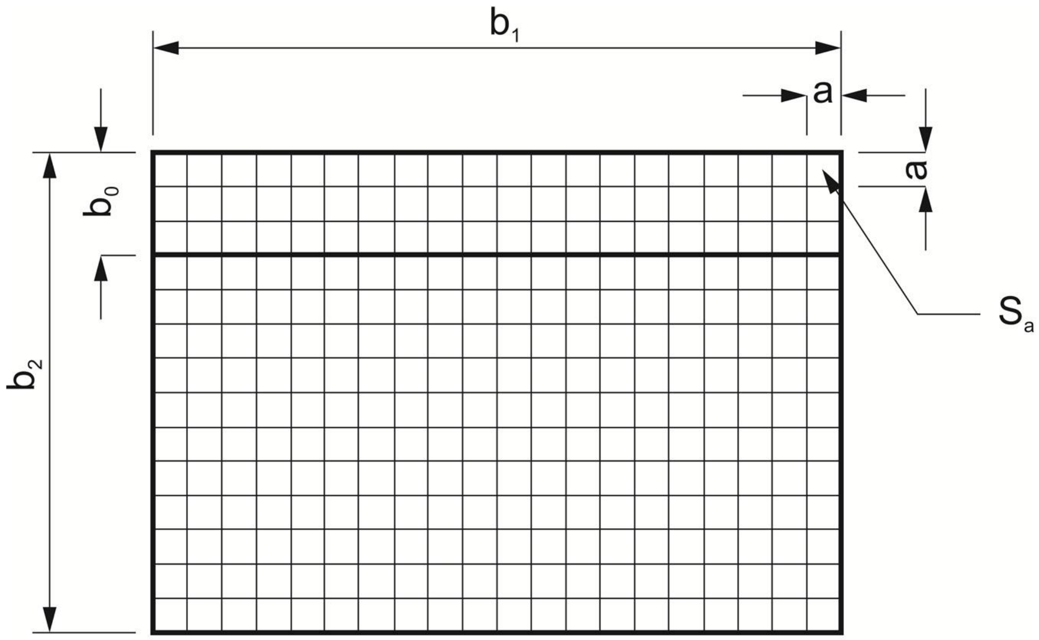

Fig. 2 Sample scheme
Source: own

To deduce the methods of evaluating coating quality, S_a elementary square size of 25 mm^2 is the basis. It can be calculated as

$$S_a = a^2 \tag{1}$$

For further calculation it is necessary to define the quality of coating eliminated on the sample within defined conditions of electroplating. Basic quality types of coatings that occur most frequently are shown in Fig. 3.

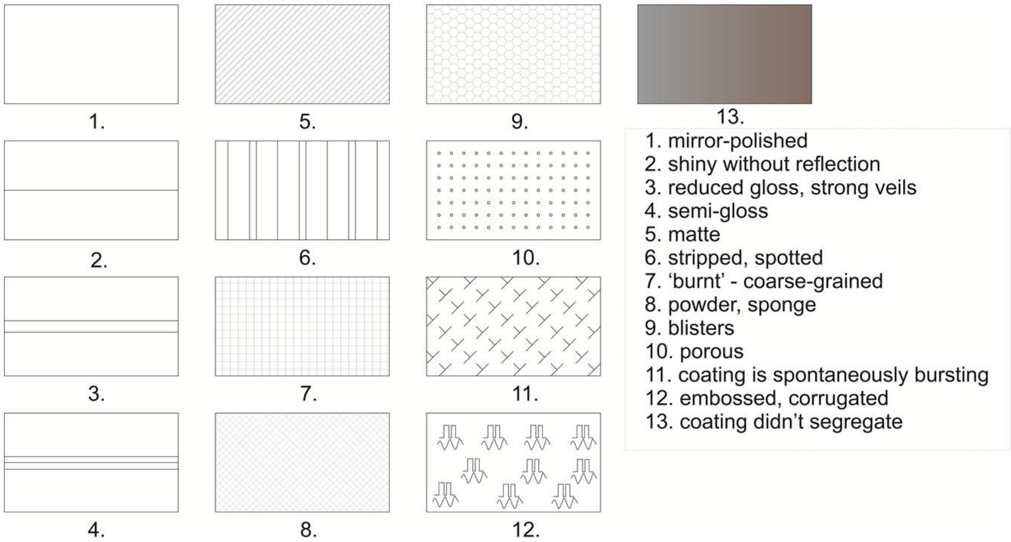

Fig. 3 Basic quality types of coatings
Source: own

Individual quality types of coatings can be characterized by their quality numbers (n_q) and scale (s_c). Description of individual quality types of eliminated coatings can be found in Tab. 3.

Quality number q_n	Description	Scale s_c
0	Not coating surface	
1	Mirror-polished	1
2	Shiny without reflection, weak veils	0,9238
3	Reduced gloss, thick veils	0,84615
4	Semi-gloss	0,76923
5	Matte	0,69231
6	Stripped, spotted	0,61538
7	'Burnt' - coarse-grained	0,53846
8	Powder, sponge	0,46154
9	Blisters	0,38462
10	Porous	0,30769
11	Coating is spontaneously bursting	0,23077
12	Embossed, corrugated	0,15385
13	Coating didn't segregate	0,07692

Tab. 3 Quality types of eliminated coatings

Consequently quality number of the type of eliminated coating is assigned to individual basic areas. The area of a particular type of eliminated coating in mm^2, which corresponds with q_n quality number, can be calculated as

$$S_{p,i} = S_a . f_i \tag{2}$$

where $i = q_n = 1,2,\ldots,13$, f_i is a number of basic areas of *i*-quality type of coating on the sample. The entire evaluated area of the sample can be obtained as it follows

$$S_H = (b_2 - b_0).b_1 = \sum_{i=1}^{13} S_{p,i} \tag{3}$$

With regard to experimental methods of forming coating in the Hull Cell, there are S_H entire evaluated area and an area on the sample, which was not electroplated. That area has zero quality number assigned. The area of non-evaluated surface of the sample can be determined as it follows

$$S_0 = b_0.b_1 = a^2 . f_{q_n=0} \tag{4}$$

The entire area of the sample is then defined as

$$S_T = b_1.b_2 = S_H + S_0 \tag{5}$$

Percentage representation of a particular *i*-quality of eliminated coating on the sample can be defined as

$$P_{H,i} = \frac{S_{p,i}}{S_T}.100 \tag{6}$$

If the value of percentage representation of particular quality of eliminated coating is multiplied by the scale of the appropriate quality of coating (Tab. 3), the total scale percentage of *i*-quality of eliminated coating on the sample

$$P_{Q,i} = P_{H,i}.s_{c,i} \tag{7}$$

The entire quality of eliminated coating can be expressed as it follows

$$Q_T = \sum_{i=1}^{13} P_{Q,i} \tag{8}$$

The result of calculation is a matrix of values, which enables the application of mathematical and statistic methods. Those methods allow us to analyze dependencies as well as predict an observed parameter – quality of eliminated coating [15].

3. RESULTS AND DISCUSSION

In general processes of surface treatment are complex multifactor systems with significant impact of mutual interactions of physical and chemical factors. Therefore observation of any parameter of formed coating at change of only one actuator often leads to incorrect results and conclusions. In the paper the results obtained by experimental verification within the electrolytic process of zinc in chloride electrolyte based on hypoboric acid were processed by DoE methods and in accordance with those methods conclusions were drawn.

The result of experiment and evaluating individual samples according to Fig. 3 and Tab. 3 is the following Tab. 4, where number of elementary evaluated areas, which correspond with individual quality types of coating, can be found for individual samples.

Sample No. / Coating Quality Type	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Sample 1	4	0	16	0	172	0	3	0	0	0	0	0	7	2
Sample 2	3	0	0	0	0	89	0	34	0	0	0	97	0	0
Sample 3	5	0	87	0	0	0	0	0	22	0	0	53	18	0
Sample 4	3	0	174	0	0	0	0	0	22	0	0	24	0	0
Sample 5	2	0	0	0	0	0	53	45	142	0	0	0	0	0
Sample 6	3	94	0	0	0	0	0	43	0	0	0	83	0	0
Sample 7	4	0	0	0	0	0	0	52	0	0	0	148	0	0
Sample 8	4	28	0	6	0	0	0	39	0	0	0	31	0	96
Sample 9	2	0	0	0	0	0	240	0	0	0	0	0	0	0
Sample 10	4	0	193	0	0	0	0	7	0	0	0	0	0	0
Sample 11	3	44	39	51	0	0	35	51	0	0	0	0	0	0
Sample 12	3	0	198	0	0	0	22	0	0	0	0	0	0	0
Sample 13	3	0	0	0	13	0	0	78	0	0	50	73	0	6
Sample 14	3	0	0	0	0	0	0	153	0	0	0	67	0	0
Sample 15	2	0	0	0	0	0	0	0	0	0	0	0	0	240
Sample 16	3	0	0	50	0	70	0	35	31	0	0	25	0	9
Sample 17	3	0	0	0	0	0	0	40	0	0	0	179	1	0
Sample 18	3	0	0	0	0	0	0	0	219	1	0	0	0	0
Sample 19	2	0	42	0	0	0	63	99	0	0	0	0	0	36
Sample 20	3	0	0	0	0	0	0	70	0	0	0	150	0	0
Sample 21	2	0	120	22	0	27	48	11	0	0	0	12	0	0
Sample 22	3	0	101	52	0	4	40	1	0	0	0	22	0	0

Tab. 4 Result of evaluating quality of coatings for individual samples (experiments)

Tab. 4 serves as the basis for application of cluster analysis. Cluster analysis helps us to identify common factors and their levels that influence the quality of formed nickel coating.

Cluster presents a group of objects which distances (dissimilarities) are smaller than distances of objects that do not belong to the cluster. According to the way of clustering procedures can be divided into hierarchical and non hierarchical clustering. Hierarchical procedures of clustering, which were used in the paper, are further divided into sintering and division clustering.

Hierarchical clustering procedures are based on hierarchical arrangement of objects and their clusters. Arranged clusters are graphically shown in the form of development tree or dendrogram. At sintering clustering two objects, which distance is the smallest one, are

connected into the first cluster and a new matrix of distances, in which objects from the first cluster are omitted, is calculated. This cluster is then classified as an object. The entire procedure is repeated so long as all objects do not form one big cluster or there remains pre given number of clusters. The procedure of division clustering is opposite. It arises from the set of all objects as the only cluster, the system of cluster is obtained by its sequential division and stage of individual objects occurs as the last one. The advantage of hierarchical methods is inutility of information about optimal number of clusters within the clustering process. Two elementary problems arise at clustering. The first one is the way of expressing similarities among objects and the second one is choice of suitable clustering procedure that has connection with selected way of expressing metrics. Basic methods of clustering metrics comprise the method of the nearest neighbour, method of the furthest neighbour, method of average separation, Ward´s method, centroid method, median method.
Hierarchical procedures with application of all elementary methods of clustering metrics were used at analysis of the process of aluminium nickel electroplating. The problem is to determine the most suitable clustering method or define the level of credibility. The first criterion to select the "best dendrogram", which corresponds with object structure and signs among objects best, is *CC* cophenetic correlation coefficient. It is Pearson correlation coefficient between real and predicted distance based on the dendogram. The higher the value of *CC*, the higher credibility and better cluster model. The second criterion of transfer closeness *Δ delta criterion*, which measures the amount of deformation of data structure earlier than degree of similarity. *Δ delta criterion* is defined as it follows

$$\Delta_A = \left[\frac{\sum_{j<k}^{N} \left| d_{jk} - d_{jk}^{*} \right|^{1/A}}{\sum_{j<k}^{N} \left(d_{jk}^{*} \right)^{1/A}} \right]^{A} \tag{9}$$

where A = 0,5 or 1, d_{ij} is distance in an original matrix of distances. It is necessary the values of Δ_A would be at about zero.

Clustering Method	CC	Delta (0.5)	Delta (1.0)
Single Linhage (Nearest Neighbour)	0,851332	0,298807	0,34608
Complete Linhage (Furthest Neighbour)	0,713927	0,23353	0,283192
Sample Average (Weighted Pain - Groups)	0,885086	0,121678	0,149404
Group Average (Uweighted Pain – Group)	0,894211	0,104676	0,126749
Median (Weighted Pain – Group Centroid)	0,86899	0,328148	0,352736
Centroid (Luweighted Pain – Group Centroid)	0,89145	0,444064	0,482905
Wanad´s Minimum Varianet – Ward´s method	0,532755	0,54132	0,583888
Flexibite strategy	0,355035	0,853529	0,881524

Tab. 5 Selection of the most suitable clustering method

Based on above mentioned theoretic assumptions and Tab. 5, the group average method can be considered to be the most suitable clustering method. At this method *CC* cophenetic correlation coefficient reaches the highest value (0,894211) and parameters Delta (0.5) and Delta (1.0) reach the lowest values. The dendrogram is shown in Fig. 4:

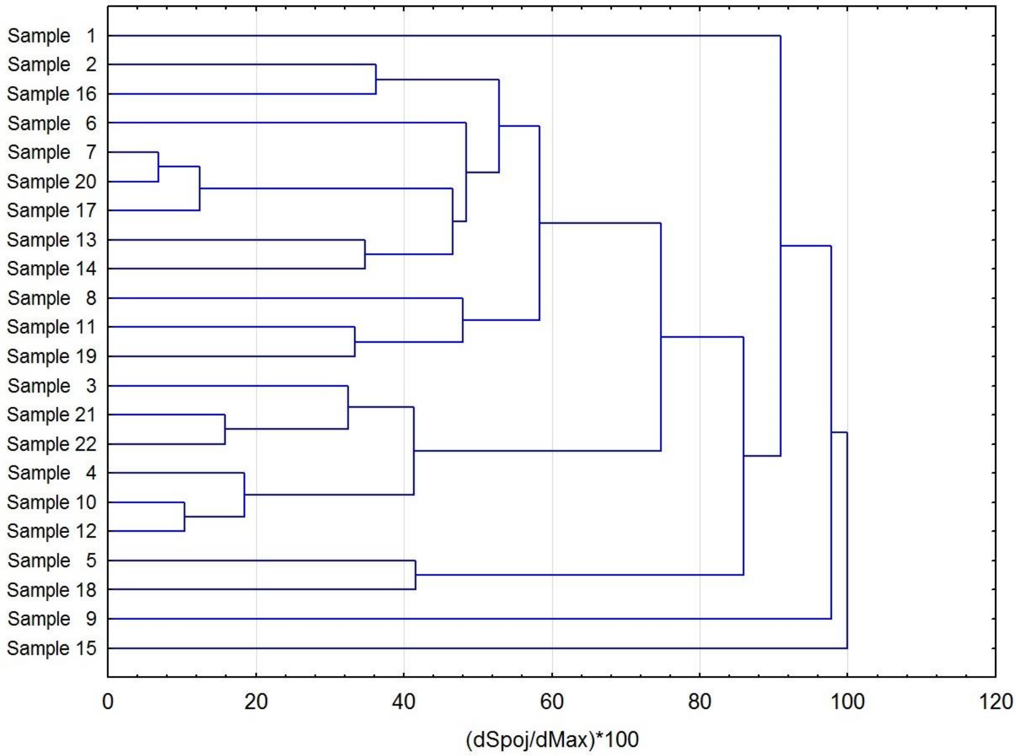

Fig. 4 Dendrogram

Based on the dendrogram two most distinctive clusters can be seen. Those are the following objects: Sample 7, Sample 20 and Sample 17 and the second cluster is formed by Sample 4, Sample 10 and Sample 12.

Fig. 5 Cluster 1
Source: own

At analysis of Cluster 1, which is formed by Sample 7, Sample 20 and Sample 17 (Fig. 5), based on the table of planned experiment (Tab. 1) it can be seen that similarity presented by the given quality type of coating can be evaluated by the entire quality of coating (8). The entire quality for Sample 7 is 31,08 %, for Sample 20 it is 32,87 % and for Sample 17 it is only 28,64 %. If experimental conditions, at which individual samples were formed, are taken into account, it can be seen that upper limit of levelling glitter additive in electrolyte (x_4) (5 ml.l^{-1}) is considered to be a common indicator. It can be assumed that increasing amount of this glitter additive causes crack of formed nickel coating. The combination of amount of nickel dichloride (x_1) in electrolyte and amount of wetting agent (x_5) in electrolyte has also impact at the combination of upper value of x_1 factor (155 g.l^{-1}) and upper value of x_5 factor (3 ml.l^{-1}) on increased coating crack.

Fig. 6 Cluster 2
Source: own

Cluster 2 represented by Sample 4, Sample 10 and Sample 12 is opposite to Cluster 1 in term of entire quality of formed layer. Sample 4 reaches the value of entire quality of coating of 80,14 %, Sample 10 reaches 90,96 % and Sample 12 reaches 89,23 %. Voltage (5 V at all 3 samples) is a common factor from the point of view of technological conditions (Tab. 1). Increase of the entire quality of nickel coating can be thus achieved by increasing voltage. It can be further seen that increase of the entire quality of eliminated coating could have been caused by the combination of high value of a basic glitter additive (x_3) and high value of levelling glitter additive (x_4) as well as the opposite combination of those two factors at lower limit of the used interval (Tab. 1).

CONCLUSION

In the paper the authors tried to point out application of cluster analysis at evaluating quality of eliminated nickel coating on aluminium samples. However, this application requires setting up a data matrix that enables meaningful utilization of that method. Correctly selected way of experimentation also creates assumptions that conclusions resulting from cluster analysis were in accordance with practical results. The way of evaluating entire quality of eliminated coating, which is proposed by the authors, enables utilization of cluster analysis and also application of other statistic methods that help to analyze influence of technological conditions of forming surface on its entire quality such as regression analysis and others.

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THE ROLE OF MATHEMATICS IN FINANCIAL EDUCATION

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Abstract: *The article analyses the results of tests of financial literacy conducted among students of the Faculty of Operation and Economics of Transport and Communications at the University of Žilina and compares them to nationwide tests. The results call for expanding the education in the field, in hope of familiarising the students with the basics of financial and insurance mathematics to the extent that would enable them to make effective decisions about personal and professional finance. Therefore a new subject, available to all study programs, is being introduced in hopes of increasing the general level of financial literacy among students.*

Keywords: financial literacy, basics of financial and insurance mathematics.

INTRODUCTION

Making qualified financial decisions has become an indispensable skill for most individuals. Yet, tests of financial literacy repeatedly point to insufficient familiarity with the basic terms.
The first extensive test of financial literacy in Slovakia has been realized in 2007 by Slovak Bank Association. The test was based on experience from tests realized abroad and on recommendations of OECD (Organisation for Economic Cooperation and Development). The results, expressed as index of financial literacy (I-FiG) ranging from 0 (minimum) to 1, described the level of ability of subjects to effectively manage personal finance.
The test contained 10 questions concerning problems related to standard accounts, loans, deposits, and investments. The average score of the index was 0.56. The score correlated positively with the level of achieved education and income, and correlated negatively with age and occupation. Among the socio-demographic groups with the highest score of I-FiG were university-educated people (I-FiG 0.66), businesspersons (I-FiG 0.65), high-school students (I-FiG 0.61). The lowest scores, in the range of 0.43-0.48, were achieved by people with basic education, unemployed and retirees. [5]

1. EDUCATION OF FINANCIAL LITERACY

In 2008, Slovak government approved a document labelled "Proposal of education strategy of finance and management of personal finance". The document has become a cornerstone for establishing National standard of financial literacy, which defines the level of knowledge and competencies expected from high-school graduates. These include basic management of personal finance, evaluating and exploiting relevant information, risk recognition, comprehension of basic terms and relationships in the financial world, setting up personal goals and upholding personal

commitments. Therein, the financial literacy is defined as "the ability to utilize information, skills, and experience to effectively manage personal resources with the aim of ensuring lifelong personal and family financial security", with an emphasis on the continuity of the process, with perpetual adaptation to personal and global economic climate. [6]
In conjunction with this philosophy, Ministry of Finance of SR introduced a web portal of financial education and customer protection, *www.fininfo.sk*, which provides and assembles advice, financial calculators, dictionary of financial terms, important documents and links to relevant legislation, test of financial literacy, as well as discussion forums. [4]
Since 2007, there have been significant improvements in the amount of information available in the public space, including mass media, internet, and a number of educative programs focused on topics in the field. Despite these improvements, the tests of financial literacy, realized as part of an SBA project "Know your money" in 2010 and 2013, yielded unsatisfactory results. [8]

2. FINANCIAL LITERACY OF FPEDAS STUDENTS

These results encouraged us to explore financial literacy of students of the Faculty of Operation and Economics of Transport and Communications (FPEDAS), who come from a wide variety of high-schools. In 2008, we conducted a similar test, with an additional goal of exploring the capabilities of students to apply basic mathematics, like calculating percentages or utilizing arithmetic and geometric series to calculate practical examples from the field of personal finance – a field essential to everyone, regardless of their relationship to mathematics. The average score of this test was 59%, in agreement with the SBA results for the population group with high-school education. [2]

2.1 Test results in 2013

In 2013, we have conducted the test again. The test group consisted of 260 participants, 80% of which were first year students.
The first question probed what financial products are used by the respondent.

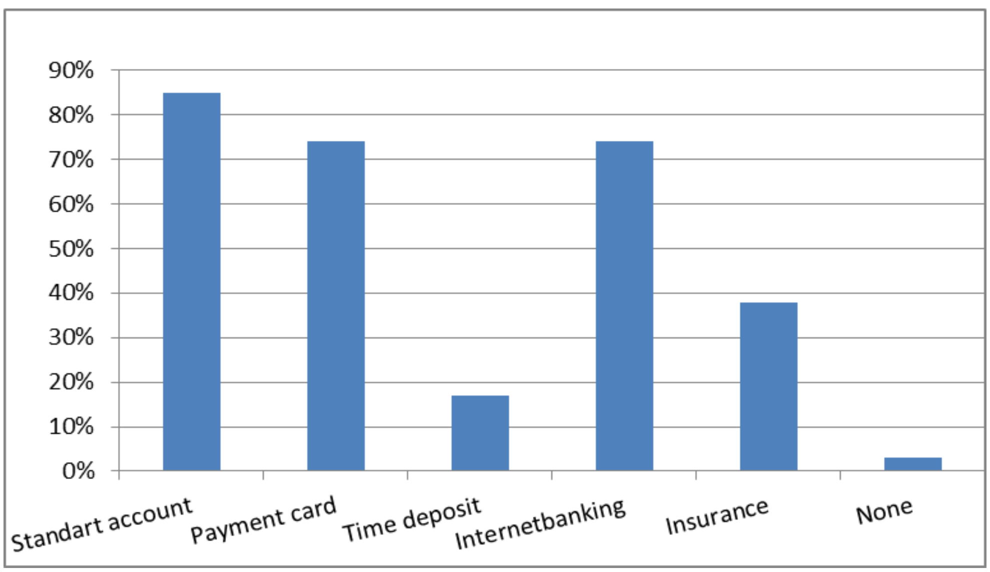

Fig. 1. Bank financial products used by respondents
Source: own

The second task aimed at indicating terms of financial vocabulary that the participant is able to comprehend and use for effective decision-making about their finance.

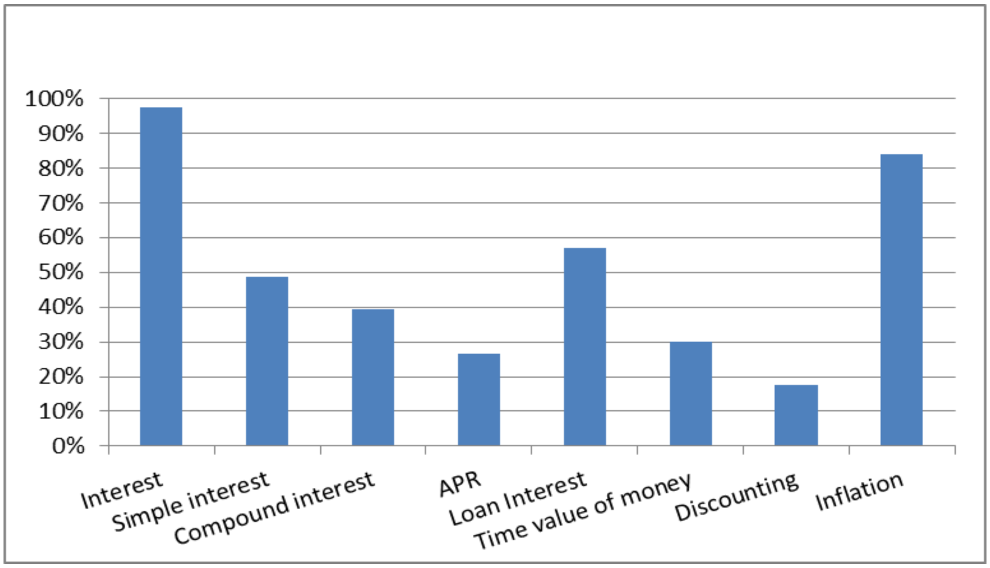

Fig. 2. Familiarity with basic terminology as indicated by the respondents**.** APR stands for annual percentage rate.
Source: own

Subsequent nine questions tested the correct use of terms like interest rates, inflation, taxation, annual percentage rate and comprehension of basic financial products like credit and debit cards, a home equity loan or building savings. Each question offered 4 choices, one of which was correct. The options included an "I don't know" as a possible answer.

Selected questions:

- Time-depositing 1000 € with 4% p.a. interest rate yields, after 2 years:

a/ more than 1080 €;
b/ 1065.85 € only, since interests are taxed by 19%;
c/ slightly more than 1065.85 €, since interests are taxed only at withdrawal;
d/ I don´t know.

- A client is offered a 30-year mortgage loan of 50,000 € with a fixed interest rate 4.01% p.a., with a monthly payment of 239 €. Pick an incorrect statement:

a/ the debt after one year is greater than 49,000 €;
b/ the debt is exactly halved after 15 years;
c/ more than 86,000 € is repaid after 30 years;
d/ I don´t know.

- Non-bank lender flyers advertise a fast loan of 10,000 € that will be due in 2 years. Which of the options is the least favourable for a client?

a/ 24 monthly payments due the last day of a month with 12% p.a.;
b/ 24 monthly payments due the last day of a month with 1% p.m.;
c/ single payment of 12,500 € after 2 years;
d/ I don´t know.

- Pick an incorrect statement:

a/ In the period 1995-2012, the average increase of price level was 5.4% p.a., which corresponds to a 2.5-fold increase of original prices;
b/ if the interest rate is 3% p.a. and the inflation is 2.8%, the overall value of the money has increased;
c/ if the interest rate is 3% p.a. and the inflation is 2.8%, the overall value of the money has decreased;
d/ I don´t know.

- A home equity loan:

a/ allows the client to purchase a car; the mortgage is insured by a local estate property;
b/ has an increased interest rate and requires no proof of income; the mortgage has to be spent on an estate property, which serves as an insurance for the mortgage at the same time;
c/ implies increased monthly payments in the early lifetime of the mortgage, due to increased risk of declining market value of a property;
d/ I don´t know.

- Pick the most favourable loan:

a/ interest rate of 7.9% p.a. monthly payment of 60 € and annual percentage rate of 15%;
b/ interest rate of 4.9% p.a. monthly payment of 55 € and annual percentage rate of 20%;
c/ I don´t know;
d/ the loans are approximately equivalent.

The full text of the test is available online [7].

2.2 Test evaluation

The results of the test are summarized in Figure 3.:

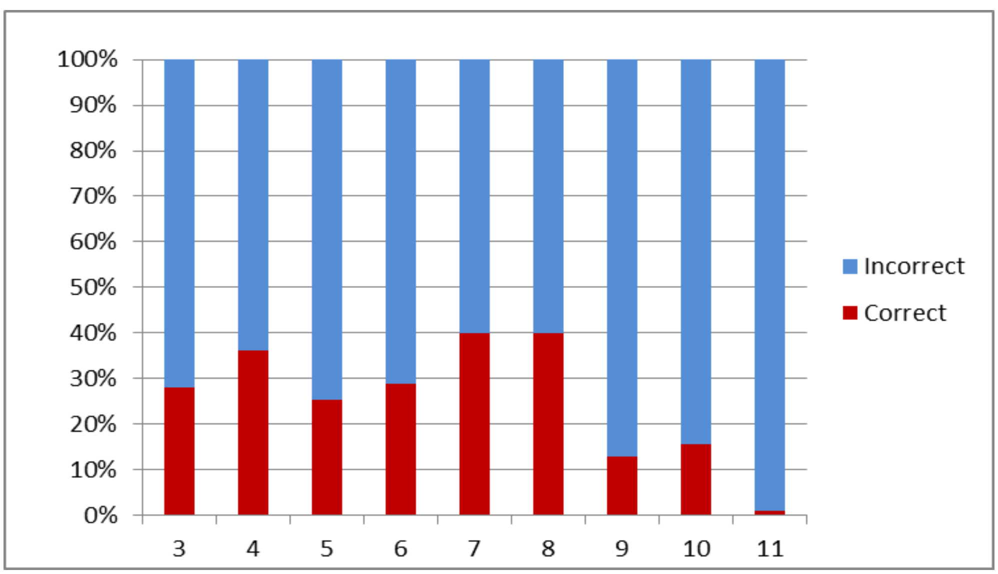

Fig. 3. Success rate of questions 3 to 11 of the test [7]
Source: own

Low success rates suggest that the students overestimate their knowledge and capabilities. Most of the students are unable to use basic financial terms and, admittedly, are incapable of even simple calculations. Despite the improved availability of information and education in the field, there has been no observable improvement.
However, the responsibility is not only on the students, but on the form and content of mathematical education on all levels, which do not motivate students sufficiently to grasp the fundamental aspects of mathematics necessary for managing personal finance.

3. FINANCIAL DECISION-MAKING IN PRACTICE

The results of the test led to proposal of a new voluntary subject *Financial decision-making in practice* for the upcoming accreditation process. The name of the subject is chosen to invoke the connection of mathematics with a topic that is indispensable to each of the students – money; yet the word „mathematics“ is deliberately avoided. The subject is targeted for 2nd-year bachelor-degree students of all programs.
The topics covered will include simple, compounded and effective interest rates, inflation, taxation of yields, real interest rates, time value of money, calculation of the present and future value of annuities, regular and non-regular annuities, loans and amortization, exchange rates and foreign exchange transactions, bills, trading of securities, introduction to life insurance and other forms of insurance, demographics, overview and the three pillars of pension system.
The subject will consist of two hours of lectures and one hour of tutorials per week, that will allow students to acquire skills necessary for effective decision-making in the field of professional and personal finance. The tutorials will be held in a computer classroom, which will enable work with online information and utilization of MS Excel for data processing.

CONCLUSION

Tests of financial literacy were conducted among students of the Faculty of Operation and Economics of Transport and Communications at the University of Žilina. In agreement with results of a nationwide study, the test revealed low level of financial literacy, as well as severe deficiency in basic mathematical education. Moreover, the awareness of these shortcomings was low, leading to overestimation of their abilities prior to the testing. Aiming to increase the level of financial literacy among the students, a new subject *Financial decision-making in practice*, available to all study programs, is being introduced at FPEDAS at the University of Žilina.

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SPC SUCH A OBJECT OF TEACHING BASIC OF PROGRAMMING

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Abstract: *The requirements of computer skills of university graduates are increasing. It is not just mastering the basic environment of the application, but also effective using their specific tools. The aim of education in this area should be the possibilities of using these tools to solve problems in practice. Students thus gain experience not only with the application itself but also with the selected issues of practice. In this contribution are shown the possibilities of teaching programming in Visual Basic for Application (VBA) for MS Excel with the connection to statistical evaluation of the achieved level of quality of the production process (SPC).*

Keywords: application, quality of production process, evaluation, VBA

INTRODUCTION

Current production practice assumes in graduates the computer skills at expert level. The role of the educational process is to prepare graduates to deal with the practical requirements. For the production process is necessary to maintain the quality requirements. For the evaluation of the achieved level serve the SPC tools. Statistical process control (SPC) is a method of quality control which uses statistical methods. SPC is used in order to monitor and control a process. In many large manufacturing organizations have available advanced information systems that are able to evaluate the achieved level of quality. These systems are often expensive for small and medium businesses, so they have to use the standard application. One possibility is to use a spreadsheet program MS Excel and its programming options -VBA. Visual Basic for Applications is Microsoft's special programming and development standard for adding functionality to programs in the Office suite. These tools are used as a support for solving various problems in engineering education [6].
In this contribution is used the basic evaluation procedures of achieved level of quality as the object for teaching programming using VBA tools.

1. STATISTICAL PROCESS CONTROL

1.1 Indices of capability

The relationship of the process specification requirements and the actual performance of the process can be described by one of several indices. Each of these indices can represent a single value that serves as measure of how well the process can produce parts or services that comply with the specification requirement [1].

$$C_p = \frac{USL - LSL}{6\sigma} \tag{1}$$

where
USL - Upper Specification Limit

LSL - Lower Specification Limit
σ - standard deviation

In a practical application, the standard deviation of process σ is almost always unknown and must be replaced by an estimate.
The C_p in equation (1) has a useful practical interpretation - the percentage of the specification band used up by the process [3].

$$P = \frac{1}{C_p}100 \tag{2}$$

$$C_{pk} = \min\left(\frac{USL-\mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right) \tag{3}$$

where μ is the average

Generally, if $C_p = C_{pk}$ the process is centered at the midpoint of the specifications, and when $C_{pk} < C_p$ the process is off-center. The magnitude of C_{pk} relative to C_p is a direct measure of how off-center the process is operating.

How is the process centred shows the index K as well. This index is to be assessed with the index C_{pk}.

$$K = \frac{|T-\mu|}{\frac{USL-LSL}{2}} \tag{4}$$

where T is the middle of the tolerance limits

1.2 Graphical interpretation

Shewharts control charts are used to detect changes in process. The simples control chart for variables data is the individual/moving range control chart. The central tendency or location of the process is monitored by tracking the average of individual subgroups averages and their behaviour relatice to a set of control limits based on the overall process average plus or minus three standard deviations [5].
The variation of the process is monitored by tracking the range of individual subgroups and their behaviour relative to a set of control limits. The control limits for averages and the ranges are based on three standard deviations. Changes in these parameters will be noted when certain rules of statistical conduct are violated [1].

Line UCL (upper control limit) and LCL (lower control limit) define the area of permissible variation values. If the process is under the statistical control about 99.7% of the values will lie within this zone.

1.2.1 Calculation of control limits

Control charts for individual measurements use the moving range of two successive observations to measure the process variability.

The moving range is defined as [4]:

$$\overline{MR} = \frac{MR_1 + MR_2 + ... + MR_m}{m} \tag{5}$$

The control limits are defined as:

$$\begin{aligned} UCL &= \mu + 3\frac{\overline{MR}}{d_2} \\ CL &= \mu \\ LCL &= \mu - 3\frac{\overline{MR}}{d_2} \end{aligned} \tag{6}$$

where μ is the average of all the individuals and $\overline{MR}$ is the average of all the moving ranges of two observations. The value of d_2 is given by specific table.

2. EVALUTION OF PROCESS USING MS EXCEL

MS Excel is one of the most widely used spreadsheet with lots of features and tools. Using VBA tools is possible to create your own applications [2]. Based on the analysis of the requirements for application have been set the basic targets of application.

The basic requirements for application are:

- load data from external file,
- graphical interpretation of achievement level of production process quality using a chart of individual measurement,
- establish the base indices describing the level of quality and their interpretation.

2.1 Using tools of Visual Basic for Application - VBA

After running of a special file that contains the VBA code is displayed the menu for selecting a file containing the input data (fig.1). This file should be saved as a file that supports macros. The program will ensure copy data from the input file into this special file.

Part of code for opening current file

```
specifyFile = Application.GetOpenFilename _
(Title:="C:\hrehova\data\value", FileFilter:="Excel Files *.xlsm (*.xlsm),")
Workbooks.Open Filename:=specifyFile
Windows("value.xlsm").Activate
```

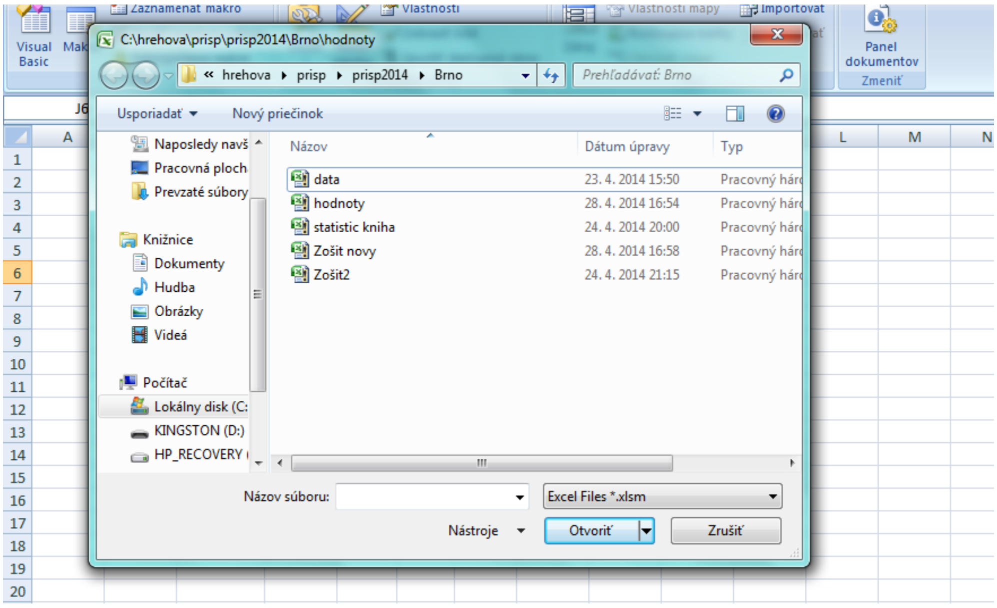

Fig. 1. Load an external data

After copying this input data is created in the main taskbar a new menu bar. This is described by the fig.2.

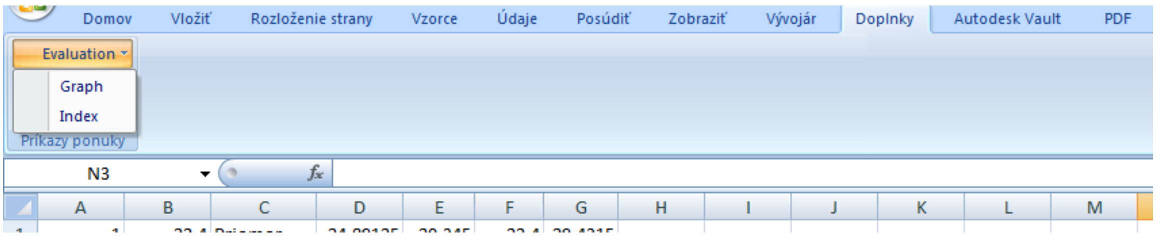

Fig. 2. Window with new menu bar

After selecting the items of menu appropriate commands are executed. The appropriate code executes a graphical evaluation of the process or calculates values of indices of process. After selection item "Index" are calculated indices based on equations (1) to (4). According to the results of these indices we can evaluate this quality. The interpretation is based on the knowledge of experts and according to the theoretical data. The interpretation is shown in the cell as well.

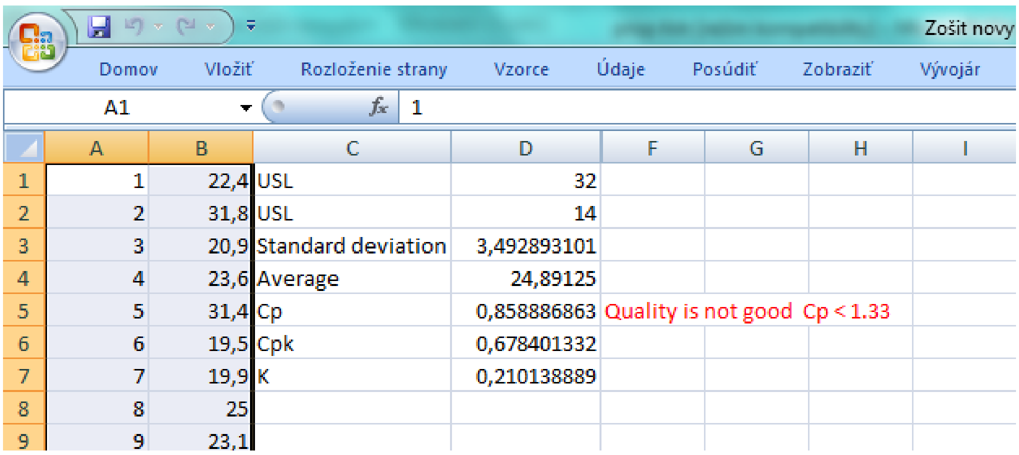

Fig. 3. Results of indices

After selection item "Graph" are calculated some controls limits, based on equations (5) and (6), and the graph is plotted (fig.4).

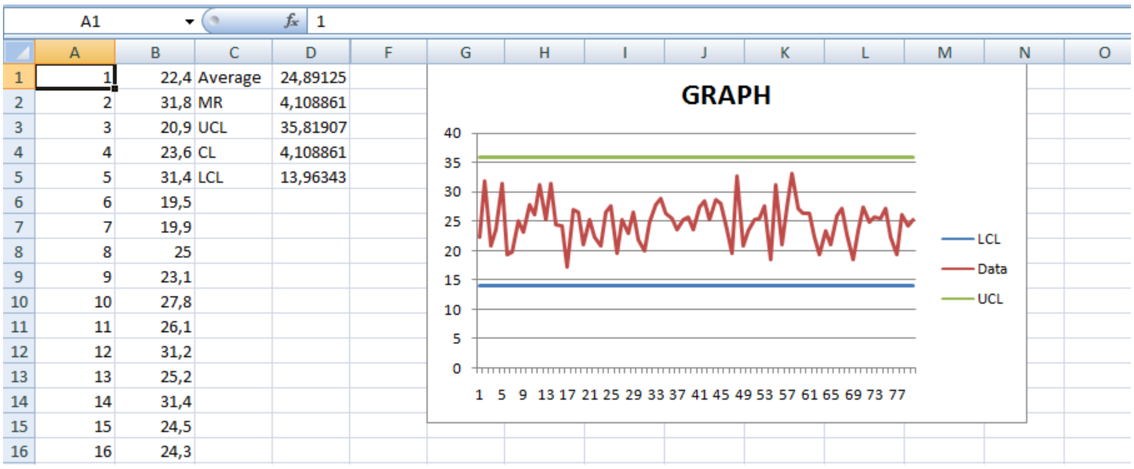

Fig. 4. Graphical interpretation of measured data

CONCLUSION

This paper describes the possibility of teaching basic programming techniques for the evaluation theoretical principles of the achieved level of quality of production process. The quality evaluation tools - SPC serve as a model for training programming commands. The students will thus acquire knowledge from two areas and gain experiences to their connection as well. The result is thus a simple application, which is also applicable for use in the practice.

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THE ICT POSSIBILITIES IN THE VIRTUAL UNIVERSITIES CYBERSPACE

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Abstract: *This paper focuses on system interpretation of applied cybernetic environment for modelling integrable adaptive educational activities in the modern cyberspace of virtual universities which link electronic business to information and knowledge. The presented vision of system integrable environment is very important due to the quick development in this area and due to production of often incompatible programmes in various current heterogeneous ICT networks. The possibilities of the existing approach to the area based on the theory of systems and the new approach to educational activities primarily from the point of view of cybernetics are described in the paper. Our own contribution to this task is currently represented by creating a system approach to the specific cyberspace solution for technical environment interface in the virtual definition of university defined in advance.*

Key words: electronic business (e-business), e-learning, cyberspace, virtual universities, information and communication technologies.

INTRODUCTION

The current specific research at the Faculty of Business and Management at the Brno University of Technologies arises from the thematic domain expressed by a new project (between 2013 and 2014) of technical, economic and legal concept of new trends in applied cybernetics.

The project basis is better defined state of knowledge in the thematic area which can globally be expressed as follows:

- The future of the culture and world civilization will primarily be linked with new intelligent models for predicting future with the needed stability and efficiency of modern and system integrable electronic environment realized by the means of information and communication technologies (ICT) in the relevant cyberspace.
- The ability to project, control and incessantly improve dynamic ICT environment and to influence rationally and in a relevant way the social environment within the modern concept of cybernetics, to react flexibly to the developing cybernetic space and to educate efficiently the social environment in the new economics of the world (i. e. in the information and knowledge society).

A new domain of information, communication, data and knowledge cyberspaces of the new economics appear which utilize unprecedented possibilities of the theory of systems, modelling, artificial intelligence and cybernetics. Generally, research and application in this dynamic domain has always been interpreted as global – gradually defined by intelligent tools created by the system solution of models, modelling and simulation of complex space-based and time-defined hierarchic structures of generally interpreted electronic business with information and knowledge (e-business), especially within the cyberspace of educational environments – generally approached in educational environments such as virtual universities. [1]

This development trend, and especially the development of electronic environment, is becoming more and more promising, forming a new area of applied cybernetics (cyberspace of the relevant social system education process and training of modern intelligent tools (ICT) and making conditions for modelling adaptive educational activities in cyberspaces of artificial intelligence defined by cybernetic safety of tools and their active protection against cyber-attacks.

1. POSSIBILITIES OF ADAPTABLE EDUCATIONAL ACTIVITIES MODELLING

This article contents represents an expression of applied cybernetic environment (cyberspace) for possible system modelling of integrable adaptive educational activities (e.g. e-learning) in the cyberspace of virtual universities combining electronic business with information and knowledge (e-business). The presented vision of the system integrable environment is extremely important from the point of view of the quick development of this area and the current production of often incompatible programmes from various heterogeneous networks of information and communication technologies (ICT).

1.1 Standard and selected development direction in the area

In the analysis of the world information sources numerous directions of development are described. Standard qualified approaches to education are described in numerous professional publications, for example [6] and [9] where knowledgeable view of broad philosophical, psychosocial, economic and educational connections are set in the European and world context together with efficiently combined managerial and lecturing skills of scientific staff of educational institutions and teachers. In other sources, for example [2] and [3], authors focus on e-learning development, combined and distant modes of studies in cooperation with enterprises and show overviews of current approaches to e-learning and blended learning while using a number of various forms and essays on the changing role of the teacher-lecturer, but also the role of the student-participant in education. It is interesting to look at e.g. [7] or [10], the area of special interest education for adult learners and evaluation of its importance for the growth of the individual and the society; with the follow-up represented by the area of modern concept of distant education from the point of view of electronic business with information and knowledge and electronic trade [5]; and also possibilities of modern tools of informatics [4] in education are of interest.

1.2 Possibilities of system definition of ICT within the cyberspace education

Every process of model making and subsequent virtual environment modelling or simulation has the character of a cybernetic system with the model feedback of the real environment of life and the virtual system. The whole identification and modelling process, or simulation and evaluation, must be performed in real time (i.e. the time when physical quantities influencing the environment can be utilized – they have their own regulatory or controlling value). That is why efficient computer systems are used nowadays to model the presented project (or simulation). The environment identification is also carried out through identifying the environment using the ICT intelligent technical tools [5], [7] and [8] (system defined in figure 1).

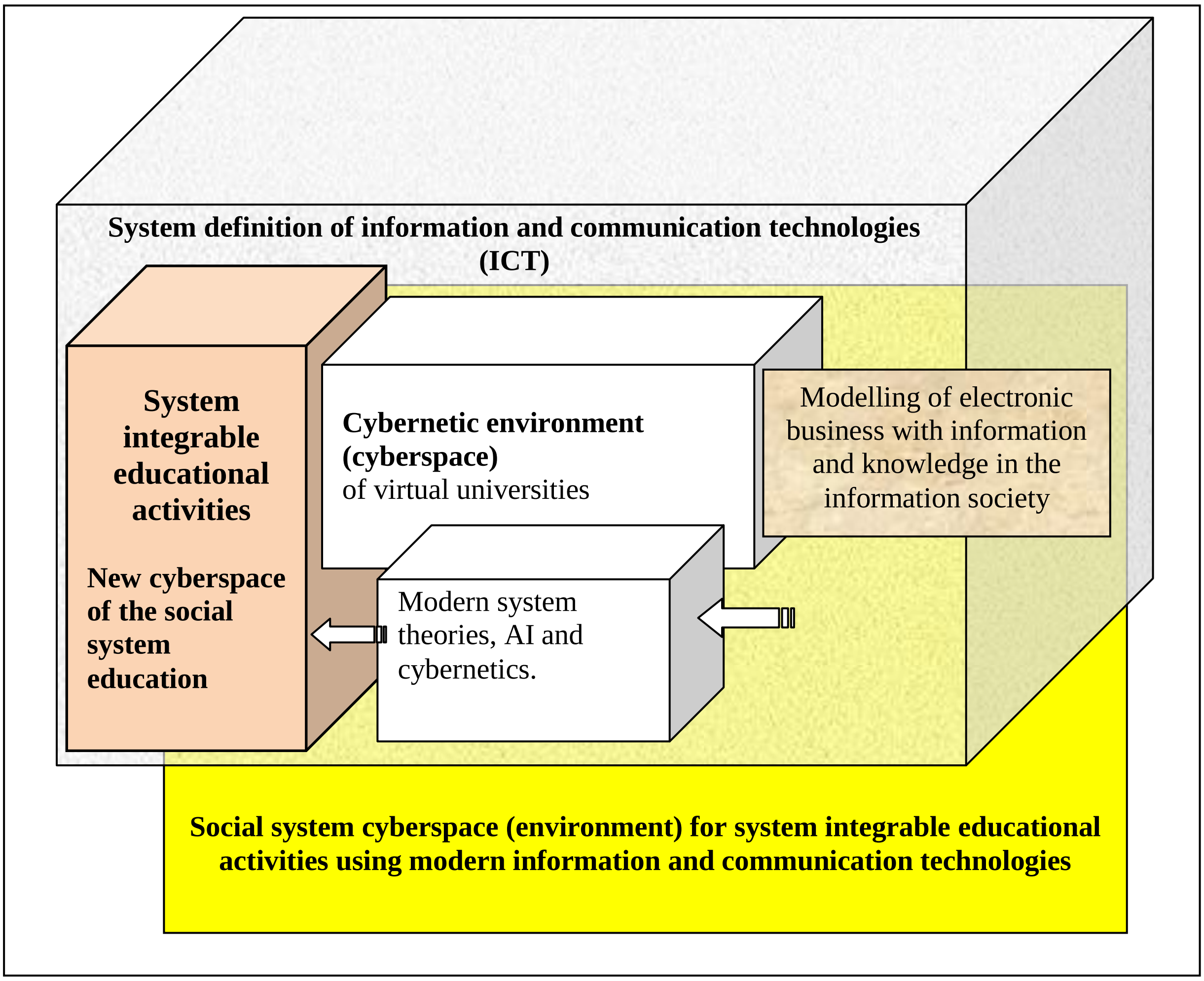

Fig. 1. System definition of cyberspace for system integrable education
Source: own

2. VIRTUAL UNIVERSITIES CYBERSPACE MODELLING

2.1 Selected options of modelling in the cyberspace of education

From the above definition of the task to solve it follows that models projected as ICT tools must be system defined and unified based on new requirements expressed through:

- Cyberspace as an immaterial sphere of information, which appears due to technical and social environments interconnection, i.e. the environment of information and communication technologies ICT enabling us to transform, store, use and interchange information (to communicate). Cyberspace must necessarily include elements of modern social system of the cultural and civilized world. Cyberspace also includes technical (hardware), programme (software) computer equipment, their database systems including the control, inputs and outputs – interconnection of communication systems with corresponding networks and the set of other devices. An example of network is the Internet, with the modern voice communication possibilities (conference, videoconference), text (e-mails), visual (skype) and other integrable social and technical interface tools [5]
- Virtual reality presented by (imaginary) domains (systems), where visitors (users) can move, communicate, observe and through their sense identify objects and models of real systems (objects) from various angles of perspective and structure, store them, analyse them and allow them to be further used by other technologies. It is possible to allow the information to be used by the quickly developing three-dimensional (3D)

printers or robotic workstations making three-dimensional useful products and models in educational process. Virtual reality (in the virtual university cyberspace) makes an impression that the user is present at the place and that he is “drawn in” the space of the imaginary environment including all his senses and functions of the body of other users. Complex perception of virtual reality requires connection of all necessary homeostasis functions (cybernetic model of humans or living nature) and suitable modern mutual connection with the virtual environment using sensors, effectors and reality simulator (simulating, for example, walking or other activity of man or living organisms),

- Behaviour and structure of cybernetic system as modern environment defining the controlling and controlled sub-system and characterizing strategy of the target behaviour – all closed with the cybernetic system feedback. Every cybernetic system in the applied form shows a number of features such as sensitivity, stability, transitional effects of the whole environment and also the level of organization in the hierarchical three-dimensional structure (the hierarchy of cybernetic systems) which is, in general features, illustrated in the cyberspace in figure 2. In applied cybernetics, there are realized digital computers and also the social system represented by elements – humans, the model of which is also a cybernetic system with internal and external feedback and the surrounding environment representing suitable interfaces in the given virtual reality and the cyberspace.
- Model and modelling is interpreted as transformation of the defined system for potential adaptation on new conditions in reality (i.e. modelling on a model). The resulting effect is on the presented research:
    - Project of the system integrable environment as a model for sharing information and knowledge in the newly interpreted social environment of virtual realities (with students in the virtual environment realized by the means of technical and programme tools suitably incorporated in the complex social and technical environment so that the new environment of the knowledge-based understanding of the process through sharing information and experience appears based on the artificial intelligence models with the aim to create intelligent (and also robotic) system at virtual university,
    - Project management plays a very important role as it gathers modern tools of adaptable educational models projecting (given by the literacy growth of the social system, the rate new information on scientific subject areas appears at, dynamics of the applied cybernetics tools – the ICT tools, implementing cyber-attacks tools in the environment of the research and the system interpreted education in the virtual universities cyberspace with the aim to enhance the overall cyber safety (cyber bullying, cyber terrorism elimination, etc.).

2.2 Selected current partial and general results of the research

The obtained materials for research are defined by:

- Detailed analysis of information sources of the world on real and estimated trends in education and the ICT dynamics and potential trends in the social systems literacy,
- Partial research materials obtained at the university (from students, teachers and other university staff),
- Partial results of modelling with PC created cyberspace,

- Creating the model of the environment with PC and obtaining partial results which are gradually published and verified in practice (and we also contribute to the assignment of bachelor and diploma theses of students, assignment of doctoral dissertation theses and new projects realized at various levels of the university).

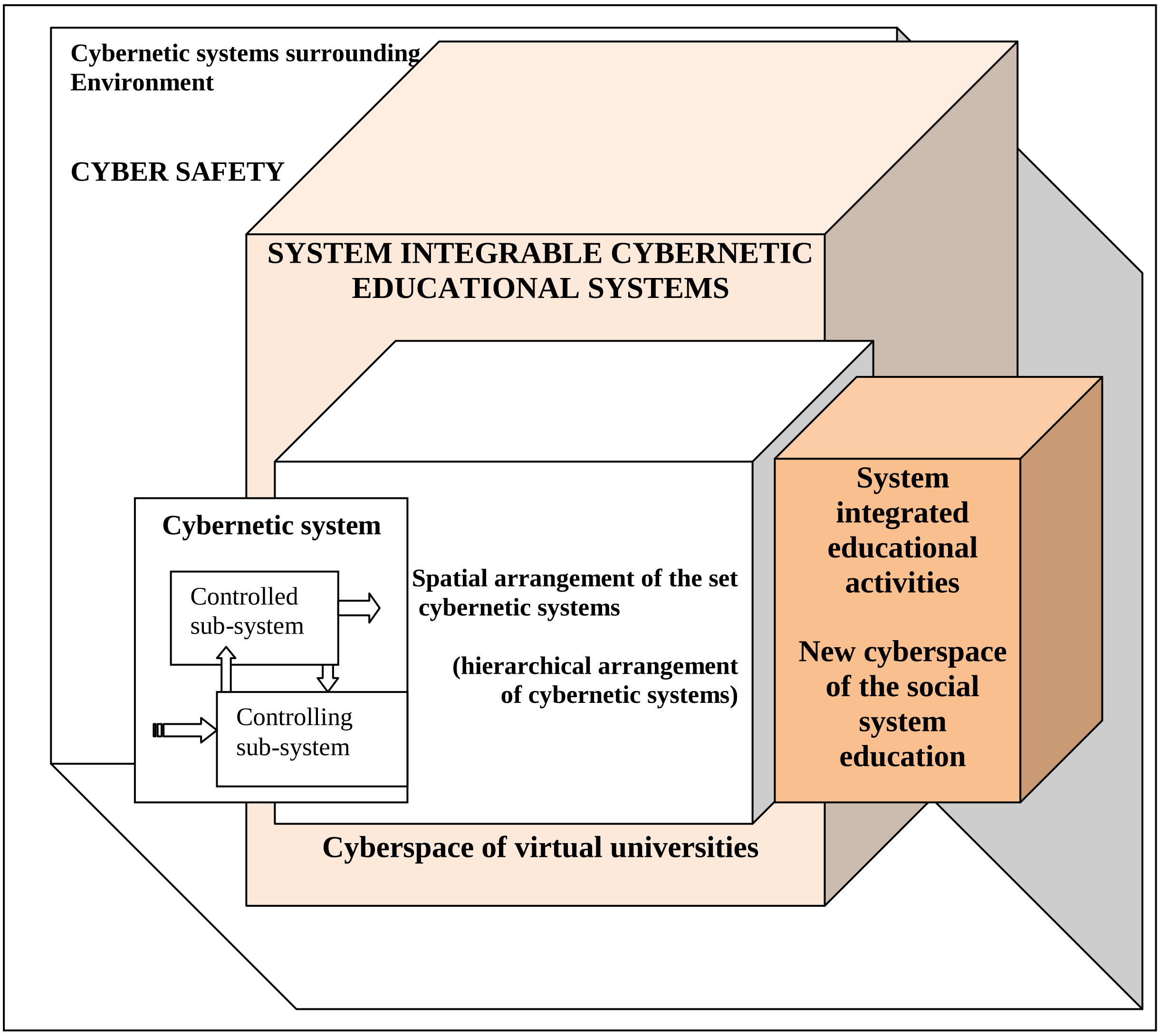

Fig. 2. Educational system cyberspace
Source: own

Partial results are interesting also from the point of view of possible:

- confirmation of the shared information rate taking into account some selected fatigue effect and other effects of students,
- connection of e-learning projects with possible economic and technical pluses and minuses, which has been verified so far,
- estimated structuralizing of teaching to micro-knowledge (and macro-knowledge) as well as their feedback connection in the verified areas, so far verified especially through electronic testing and initial solving of some selected practical applications,
- description of the emerging methodology of educational activities modern projecting in the virtual university cyberspace.

CONCLUSION

There are several partial solution levels in the specific research. Currently, we have contributed to this task by creating system approach to specific cyberspace solution (schematically demonstrated in figures 1 and 2) and by simplified mathematic model that has been created in the virtual definition of university and environment trends defined with difficulties, which are given especially by the development of the social and technical environment for the new ICT. The research further continues in the area of modern projecting and project management of adaptable educational tools.

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Stability of nonlinear difference systems, which describes the dynamics of neutral networks

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Abstract:

In the contribution is considered stability of a planar discrete system of equations with weak nonlinearities. Criteria of asymptotic stability of an equilibrium point are developed.

Keywords: Stability, nonlinear difference systems, plane model, equilibrium.

Introduction

The modern theory of dynamical neural networks has two directions. The first is a neural network training, i.e. determining the parameters of the network. The second is a direction of its operation, i.e. recognition process is sent a signal sequence and determining the membership of input trained.
The second direction may be either static, i.e. an ”instant”determine whether the signal sent by a trained or ”adaptive”, i.e. determining compliance in the process

of some iterative procedure. It can be formalized as the dynamics of a system of difference equations. If the process converges asymptotically to a steady, appropriate training, the neural network is functioning properly.

Iterative processes in neural networks made it necessary for studying the stability of a stationary equilibrium position, located in the first quadrant of nonlinear systems with a diagonal linear part and the convergence to a stationary equilibrium position [1, p.853]. In [2] the author considers mathematical models of dynamic neural networks with delay. Apparatus for studying is based on the second method of Lyapunov functionals Lyapunov-Krasovskii. In [3], the stability of the equilibrium position of delay systems was investigated using majorant methods.

Investigation of the stability of linear systems with time delay was carried out in [4,5]. In this paper we consider nonlinear planar systems with a "weak nonlinearity". By "weak nonlinearity"we mean that the nonlinear functions satisfy the Lipschitz condition with a "small"constant. We investigate the stability of stationary solutions of delay systems using the method of Lyapunov functions with the additional Razumikhin condition [6].

1 Planar system without delay

Consider a neural system, whose dynamics is described by a system of two nonlinear difference equations.

$$\begin{aligned} y_1(k+1) &= a_{11}y_1(k) + f_{11}(y_1(k)) + f_{12}(y_2(k)) + b_1, \\ y_2(k+1) &= a_{22}y_2(k) + f_{21}(y_1(k)) + f_{22}(y_2(k)) + b_2. \end{aligned} \tag{1.1}$$

Here $a_{11},\ a_{22}$ are constants, $f_{ij},\ i,j = \overline{1,2}$ are nonlinearities satisfyin Lipschitz condition with constant $L_{ij},\ i,j = \overline{1,2}$

$$\mid f_{ij}(y_j + \Delta y_j) - f_{ij}(y_j) \mid \leq L_{ij} \mid \Delta y_j \mid,\ i,j = \overline{1,2}. \tag{1.2}$$

Assume that the system of equations

$$\begin{aligned} (1-a_{11})y_1 - f_{11}(y_1) - f_{12}(y_2) &= b_1, \\ (1-a_{22})y_2 - f_{21}(y_1) - f_{22}(y_2) &= b_2. \end{aligned} \tag{1.3}$$

has a unique solution $M_0(y_1^0, y_2^0),\ y_1^0 > 0,\ y_2^0 > 0$. Make the change of variables $y_1(k) = x_1(k) + y_1^0,\ y_2(k) = x_2(k) + y_2^0$. After substituting in (2.1) we obtain

$$\begin{aligned} x_1(k+1) + y_1^0 &= a_{11}(x_1(k) + y_1^0) + f_{11}(x_1(k) + y_1^0) + f_{12}(x_2(k) + y_2^0) + b_1, \\ x_2(k+1) + y_2^0 &= a_{22}(x_2(k) + y_2^0) + f_{21}(x_1(k) + y_1^0) + f_{22}(x_2(k) + y_2^0) + b_2. \end{aligned} \tag{1.4}$$

Rewrite the resulting system (2.4) in the form

$$
\begin{aligned}
x_1(k+1) = a_{11}x_1(k) + & \\
& + \left| \, f_{11}(x_1(k)+y_1^0) - f_{11}(y_1^0) \, \right| + \\
& + \left| \, f_{12}(x_2(k)+y_2^0) - f_{12}(y_2^0) \, \right| - \\
& \qquad - \left| \, (1-a_{11})y_1^0 - f_{11}(y_1^0) - f_{12}(y_2^0) - b_1 \, \right|,
\end{aligned}
$$

$$
\begin{aligned}
x_2(k+1) = a_{22}x_2(k) + & \\
& + \left| \, f_{21}(x_1(k)+y_1^0) - f_{21}(y_1^0) \, \right| + \\
& + \left| \, f_{22}(x_2(k)+y_2^0) - f_{22}(y_2^0) \, \right| - \\
& \qquad - \left| \, (1-a_{22})y_1^0 - f_{21}(y_1^0) - f_{22}(y_2^0) - b_2 \, \right| .
\end{aligned}
$$

Let

$$
\begin{aligned}
F_{11}(x_1(k)) &= f_{11}(x_1(k)+y_1^0) - f_{11}(y_1^0), \\
F_{12}(x_2(k)) &= f_{12}(x_2(k)+y_2^0) - f_{12}(y_2^0), \\
F_{21}(x_1(k)) &= f_{21}(x_1(k)+y_1^0) - f_{21}(y_1^0), \\
F_{22}(x_2(k)) &= f_{22}(x_2(k)+y_2^0) - f_{22}(y_2^0).
\end{aligned}
$$

Using (2.3), we find that the system (2.4) takes the form

$$
\begin{aligned}
x_1(k+1) &= a_{11}x_1(k) + F_{11}(x_1(k)) + F_{12}(x_2(k)) \\
x_2(k+1) &= a_{22}x_2(k) + F_{21}(x_1(k)) + F_{22}(x_2(k)).
\end{aligned} \tag{1.5}
$$

Because

$$
F_{11}(0) = 0, \; F_{12}(0) = 0, \; F_{21}(0) = 0, \; F_{22}(0) = 0,
$$

the study of the stability of the equilibrium position $M_0(y_1^0, y_2^0)$ system (2.1) has been reduced to the study of the stability of the zero equilibrium state of the system (2.5).

Theorem 1 *Let the system* (2.3) *have a unique solution* $M_0(y_1^0, y_2^0), \; y_1^0 > 0, \; y_2^0 > 0$ *and let there exist constants* $h_{11} > 0, \; h_{22} > 0,$ *such that*

$$
c_{11} > 0, \; c_{12}c_{22} - c_{12}^2 > 0, \tag{1.6}
$$

where

$$\begin{aligned} c_{11} &= h_{11}(1-a_{11}^2) - 2h_{11} \mid a_{11} \mid L_{11} - h_{11}L_{11}^2 - h_{22}L_{21}^2, \\ c_{12} &= -h_{11}L_{12}(\mid a_{11} \mid + L_{11}) - h_{22}L_{21}(\mid a_{22} \mid + L_{22}), \\ c_{22} &= h_{22}(1-a_{22}^2) - 2h_{22} \mid a_{11} \mid L_{22} - h_{22}L_{22}^2 - h_{11}L_{12}^2. \end{aligned} \tag{1.7}$$

Then the equilibrium $M_0(y_1^0, y_2^0)$ *system* (2.1) *is asymptotically stable.*

Proof. To investigate the stability of the equilibrium position $M_0(y_1^0, y_2^0)$ use a quadratic Lyapunov function of the form $V(x_1, x_2) = h_{11}x_1^2 + h_{22}x_2^2$. Its first difference along the trajectories of the system (2.6) has the form

$$\Delta V(x_1(k), x_2(k)) = [h_{11}x_1^2(k+1) + h_{22}x_2^2(k+1)] - [h_{11}x_1^2(k) + h_{22}x_2^2(1),$$

or

$$\begin{aligned} &\Delta V(x_1(k), x_2(k)) = \\ &\qquad = h_{11}[a_{11}x_1(k) + F_{11}(x_1(k)) + F_{12}(x_2(k))]^2 + \\ &\qquad + h_{22}[a_{22}x_2(k) + F_{21}(x_1(k)) + F_{22}(x_2(k))]^2 - \\ &\qquad\qquad - [h_{11}x_1^2(k) + h_{22}x_2^2(1)]. \end{aligned}$$

Transform the expression obtained by selecting the quadratic components, as follows:

$$\begin{aligned} &\Delta V(x_1(k), x_2(k)) = \\ &\qquad = h_{11}[a_{11}x_1(k) + F_{11}(x_1(k)) + F_{12}(x_2(k))]^2 + \\ &\qquad + h_{22}[a_{22}x_2(k) + F_{21}(x_1(k)) + F_{22}(x_2(k))]^2 - \\ &\qquad\qquad - [h_{11}x_1^2(k) + h_{22}x_2^2(1)]. \end{aligned}$$

$$\begin{aligned} &\Delta V(x_1(k), x_2(k)) = -h_{11}(1-a_{11}^2)x_1^2(k) - h_{22}(1-a_{22})x_2^2(k) + \\ &+ h_{11}\{2a_{11}x_1(k)[F_{11}(x_1(k)) + F_{12}(x_2(k))] + [F_{11}(x_1(k)) + F_{12}(x_2(k))]^2\} + \\ &+ h_{22}\{2a_{22}x_2(k)[F_{21}(x_1(k)) + F_{22}(x_2(k))] + [F_{21}(x_1(k)) + F_{22}(x_2(k))]^2\}. \end{aligned}$$

Using the Lipschitz condition (2.2) imposed on the functions $F_{ij}(.)$ we can rewrite this expression as follows:

$$\begin{aligned} &\Delta V(x_1(k), x_2(k)) \le -h_{11}(1-a_{11}^2)x_1^2(k) - h_{22}(1-a_{22})x_2^2(k) + \\ &+ 2h_{11}|a_{11}||x_1(k)|[L_{11}|x_1(k)| + L_{12}|x_2(k)|] + h_{11}[L_{11}|x_1(k)| + L_{12}|x_2(k)|]^2 + \\ &+ 2h_{22}|a_{22}||x_2(k)|[L_{21}|x_1(k)| + L_{22}|x_2(k)|] + h_{22}[L_{11}|x_1(k)| + L_{22}|x_2(k)|]^2. \end{aligned}$$

Now represent the right side as a quadratic form

$$\begin{aligned}\Delta V(x_1(k), x_2(k)) \le -[h_{11}(1-a_{11}^2) - 2h_{11}|a_{11}|L_{11} - h_{11}L_{11}^2 - h_{22}L_{21}^2]x_1^2(k) +\\ + 2h_{11}[h_{11}L_{12}(|a_{11}| + L_{11}) + h_{22}L_{21}(|a_{22}| + L_{22})]|x_1(t)||x_2(t)| -\\ - [h_{22}(1-a_{22}^2) - 2h_{22}|a_{22}|L_{22} - h_{22}L_{22}^2 - h_{11}L_{12}^2]x_2^2(k).\end{aligned}$$

Resulting inequality can be written in the form

$$\Delta V(x_1(k), x_2(k)) \le -z^T(k)Cz(k), z(k) = \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix}, C = \begin{pmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{pmatrix}$$

$$\begin{aligned} c_{11} &= h_{11}(1-a_{11}^2) - 2h_{11} \mid a_{11} \mid L_{11} - h_{11}L_{11}^2 - h_{22}L_{21}^2, \\ c_{12} &= -h_{11}L_{12}(\mid a_{11} \mid + L_{11}) - h_{22}L_{21}(\mid a_{22} \mid + L_{22}), \\ c_{22} &= h_{22}(1-a_{22}^2) - 2h_{22} \mid a_{11} \mid L_{22} - h_{22}L_{22}^2 - h_{11}L_{12}^2. \end{aligned}$$

Using the Sylvester criterion, the condition of negative definiteness of the Lyapunov function, i.e., condition of asymptotic stability of the equilibrium of the system (2.5) has the form (2.6).

Remark 1 *It follows from* (2.6) *that a necessary condition for asymptotic stability of solutions to the equilibrium position is the "smallness"of the nonlinear terms, i.e. Lipschitz constant.*

Indeed, from (2.6) we see that it is necessary to satisfy the inequalities

$$a_{11}^2 + 2|a_{11}|L_{11} + L_{11}^2 + \frac{h_{22}}{h_{11}}L_{21}^2 < 1, a_{22}^2 + 2|a_{22}|L_{22} + L_{22}^2 + \frac{h_{11}}{h_{22}}L_{12}^2 < 1.$$

2 Planar system with delays

Consider the system on the plane with delays

$$\begin{aligned} y_1(k+1) &= a_{11}y_1(k) + f_{11}(y_1(k-m_{11})) + f_{12}(y_2(k-m_{12})) + b_1, \\ y_2(k+1) &= a_{22}y_2(k) + f_{21}(y_1(k-m_{21})) + f_{22}(y_2(k-m_{22})) + b_2 \end{aligned} \tag{2.1}$$

where delays m_{ij} are natural numbers, $i, j = 1, 2$ and functions $f_{ij},\ i, j = \overline{1, 2}$ satisfy the Lipschitz condition (2.2). As for the system without delay, make the change $y_1(k) = x_1(k) + y_1^0,\ y_2(k) = x_2(k) + y_2^0$ and the system (3.1) reduces to the system

$$x_1(k+1) + y_1^0 =$$
$$a_{11}(x_1(k) + y_1^0) + f_{11}(x_1(k - m_{11}) + y_1^0) + f_{12}(x_2(k - m_{12}) + y_2^0) + b_1,$$
$$x_2(k+1) + y_2^0 =$$
$$a_{22}(x_2(k) + y_2^0) + f_{21}(x_1(k - m_{21}) + y_1^0) + f_{22}(x_2(k - m_{22}) + y_2^0) + b_2.$$

Rewrite it as

$$\begin{aligned} x_1(k+1) &= a_{11}x_1(k) + F_{11}(x_1(k - m_{11})) + F_{12}(x_2(k - m_{12})), \\ x_2(k+1) &= a_{22}x_2(k) + F_{21}(x_1(k - m_{21})) + F_{22}(x_2(k - m_{22})) \end{aligned} \tag{2.2}$$

where

$$\begin{aligned} F_{11}(x_1(k - m_{11})) &= f_{11}(x_1(k - m_{11}) + y_1^0) - f_{11}(y_1^0), \\ F_{12}(x_2(k - m_{12})) &= f_{12}(x_2(k - m_{12}) + y_2^0) - f_{12}(y_2^0), \\ F_{21}(x_1(k - m_{21})) &= f_{21}(x_1(k - m_{21}) + y_1^0) - f_{21}(y_1^0), \\ F_{22}(x_2(k - m_{22})) &= f_{22}(x_2(k - m_{22}) + y_2^0) - f_{22}(y_2^0). \end{aligned}$$

As for systems without delay, the study of stability of the equilibrium position $M_0(y_1^0, y_2^0)$ of system (3.1) has been reduced to the study of the stability of the zero equilibrium state of the system (3.2). We give conditions for asymptotic stability.

Theorem 2 *Let the system* (2.3) *have a unique solution* $M_0(y_1^0, y_2^0)$ *and let there exist constants* $h_{11} > 0,\ h_{22} > 0$, *such that*

$$h_{11}(1 - a_{11}^2) - h_{11}K_{11} - h_{22}K_{12} > 0,\ h_{22}(1 - a_{22}^2) - h_{11}K_{21} - h_{22}K_{22} > 0, \tag{2.3}$$

where

$$K_{11} = |a_{11}|\left(2L_{11} + L_{12}\left(1 + \frac{h_{11}}{h_{22}}\right)\right) + L_{11}^2 + L_{11}L_{12}\left(1 + \frac{h_{11}}{h_{22}}\right) + L_{12}^2\frac{h_{11}}{h_{22}},$$

$$K_{12} = |a_{22}|\left(L_{21} + L_{22}\frac{h_{11}}{h_{22}}\right) + L_{21}^2 + L_{21}L_{22}\left(1 + \frac{h_{11}}{h_{22}}\right) + L_{22}^2\frac{h_{11}}{h_{22}},$$

$$K_{21} = |a_{11}|\left(L_{11}\frac{h_{22}}{h_{11}} + L_{12}\right) + L_{11}^2\frac{h_{22}}{h_{11}} + L_{11}L_{12}\left(1 + \frac{h_{22}}{h_{11}}\right) + L_{12}^2,$$

$$K_{22} = |a_{22}|\left(L_{21}\left(1 + \frac{h_{22}}{h_{11}}\right) + 2L_{22}\right) + L_{21}^2\frac{h_{22}}{h_{11}} + L_{21}L_{22}\left(1 + \frac{h_{22}}{h_{11}}\right) + L_{22}^2.$$

Then the equilibrium $M_0(y_1^0, y_2^0)$ *is asymptotically stable.*

Proof. In the study of stability of the equilibrium $M_0(y_1^0, y_2^0)$ we will again use the quadratic Lyapunov function $V(x_1, x_2) = h_{11}x_1^2 + h_{22}x_2^2$, in the calculation of the first difference of the Lyapunov function by virtue of (3.2) we use the Razumikhin condition, modified for the difference systems [6,7]. Razumikhin condition has a simple geometric interpretation. It means that for the first difference of the Lyapunov function along the solutions of the system is calculated under the condition that prehistory is within the level of the corresponding surface of the Lyapunov function for the function $V(x_1, x_2) = h_{11}x_1^2 + h_{22}x_2^2$, it has the form

$$\begin{aligned} h_{11}x_1^2(k-m_{ij}) + h_{22}^2x_2^2(k-m_{ij}) = V(x_1(k-m_{ij}), x_2(k-m_{ij})) < \\ < V(x_1(k), x_2(k)) = h_{11}x_1^2(k) + h_{22}^2x_2^2(k), m_{ij} > 0, i, j = 1, 2. \end{aligned} \tag{2.4}$$

It follows that

$$|x_1(k-m)| < \sqrt{x_1^2(k) + \frac{h_{22}}{h_{11}}x_2^2(k)}, |x_2(k-m)| < \sqrt{\frac{h_{11}}{h_{22}}x_1^2(k) + x_2^2(k)}. \tag{2.5}$$

The first difference of the Lyapunov function $V(x_1, x_2)$ by virtue of (3.2) has the form

$$\begin{aligned} \Delta V(x_1(k), x_2(k)) = -h_{11}(1-a_{11})x_1^2(k) + h_{22}(1-a_{22})x_2^2(k) \\ + h_{11}\{2a_{11}x_1(k)[F_{11}(x_1(k-m_{11})) + F_{12}(x_2(k-m_{12}))]\} \\ + h_{11}\{[F_{11}(x_1(k-m_{11})) + F_{12}(x_2(k-m_{12}))]^2\} \\ + h_{22}\{2a_{22}x_2(k)[F_{21}(x_1(k-m_{21})) + F_{22}(x_2(k-m_{22}))]\} \\ + h_{22}\{[F_{21}(x_1(k-m_{21})) + F_{22}(x_2(k-m_{22}))]^2\}. \end{aligned}$$

$$\begin{aligned} \Delta V(x_1(k), x_2(k)) = -h_{11}(1-a_{11}^2)x_1^2(k) - h_{22}(1-a_{22})x_2^2(k) \\ + h_{11}\{2a_{11}x_1(k)[F_{11}(x_1(k)) + F_{12}(x_2(k))] + [F_{11}(x_1(k)) + F_{12}(x_2(k))]^2\} \\ + h_{22}\{2a_{22}x_2(k)[F_{21}(x_1(k)) + F_{22}(x_2(k))] + [F_{21}(x_1(k)) + F_{22}(x_2(k))]^2\}. \end{aligned}$$

Using the Lipschitz condition (2.2), we obtain

$$\begin{aligned} \Delta V(x_1(k), x_2(k)) \leq -h_{11}(1-a_{11}^2)x_1^2(k) - h_{22}(1-a_{22})x_2^2(k) + \\ + 2h_{11}|a_{11}||x_1(k)|[L_{11}|x_1(k-m_{11})| + L_{12}|x_2(k-m_{12})|] \\ + h_{11}[L_{11}|x_1(k-m_{11})| + L_{12}|x_2(k-m_{12})|]^2 + \\ + 2h_{22}|a_{22}||x_2(k)|[L_{21}|x_1(k-m_{21})| + L_{22}|x_2(k-m_{22})|] \\ + h_{22}[L_{11}|x_1(k-m_{21})| + L_{22}|x_2(k-m_{22})|]^2. \end{aligned}$$

or

$$\begin{aligned}
\Delta V(x_1(k),x_2(k)) &\le -h_{11}(1-a_{11}^2)x_1^2(k) - h_{22}(1-a_{22})x_2^2(k) + \\
&+ 2h_{11}|a_{11}|L_{11}|x_1(k)||x_1(k-m_{11})| + 2h_{11}|a_{11}|L_{12}|x_1(k)||x_2(k-m_{12})| \\
&+ h_{11}[L_{11}^2|x_1(k-m_{11})|^2 + 2h_{11}L_{11}L_{12}|x_1(k-m_{11})||x_2(k-m_{12})| \\
&+ h_{11}[L_{12}^2|x_2(k-m_{12})|^2 + 2h_{22}|a_{22}|L_{21}|x_2(k)||x_1(k-m_{21})| \\
&+ 2h_{22}|a_{22}|L_{22}|x_2(k)||x_2(k-m_{22})| + h_{22}[L_{21}^2|x_1(k-m_{21})|^2 \\
&+ 2h_{21}L_{22}L_{12}|x_1(k-m_{21})||x_2(k-m_{22})| + h_{22}[L_{22}^2|x_2(k-m_{22})|^2.
\end{aligned}$$

Applying Razumikhin condition (3.5) and the inequality

$$2AB \le A^2 + B^2, \tag{2.6}$$

we obtain

$$2|x_1(k)||x_1(k-m_{ij})| < 2|x_1(k)|\sqrt{x_1^2(k) + \frac{h_{22}}{h_{11}}x_2^2(k)} \le \left[2|x_1^2(k) + \frac{h_{22}}{h_{11}}x_2^2(k)\right],$$

$$2|x_1(k)||x_2(k-m_{ij})| < 2|x_1(k)|\sqrt{\frac{h_{11}}{h_{22}}x_1^2(k) + x_2^2(k)} \le \left[\left(1+\frac{h_{11}}{h_{22}}\right)x_1^2(k) + x_2^2(k)\right],$$

$$2|x_2(k)||x_1(k-m_{ij})| < 2|x_2(k)|\sqrt{x_1^2(k) + \frac{h_{22}}{h_{11}}x_2^2(k)} \le \left[x_1^2(k) + \left(1+\frac{h_{22}}{h_{11}}\right)x_2^2(k)\right],$$

$$2|x_2(k)||x_2(k-m_{ij})| < 2|x_2(k)|\sqrt{\frac{h_{11}}{h_{22}}x_1^2(k) + x_2^2(k)} \le \left[\frac{h_{11}}{h_{22}}x_1^2(k) + 2x_2^2(k)\right],$$

$$2|x_1(k-m_{ij})||x_2(k-m_{ij})| < 2\sqrt{x_1^2(k) + \frac{h_{22}}{h_{11}}x_2^2(k)} \times \sqrt{\frac{h_{11}}{h_{22}}x_1^2(k) + x_2^2(k)} \le$$

$$\le \left[\left(1+\frac{h_{11}}{h_{22}}\right)x_1^2(k) + \left(1+\frac{h_{22}}{h_{11}}\right)x_2^2(k)\right],$$

$$x_1^2(k-m_{ij}) < x_1^2(k) + \frac{h_{22}}{h_{11}}x_2^2(k), x_2^2(k-m_{ij}) < \frac{h_{11}}{h_{22}}x_1^2(k) + x_2^2(k). \tag{2.7}$$

Hence, using (3.7), we obtain that

$$\Delta V(x_1(k), x_2(k)) \leq -h_{11}(1-a_{11}^2)x_1^2(k) - h_{22}(1-a_{22})x_2^2(k)+$$
$$+h_{11}|a_{11}|L_{11}\left[2x_1^2(k)+\frac{h_{22}}{h_{11}}x_2^2(k)\right]+h_{11}|a_{11}|L_{12}\left[\left(1+\frac{h_{11}}{h_{22}}\right)x_1^2(k)+x_2^2(k)\right]+$$
$$+h_{11}L_{11}^2\left[x_1^2(k)+\frac{h_{22}}{h_{11}}x_2^2(k)\right]+h_{11}L_{11}L_{12}\left[\left(1+\frac{h_{11}}{h_{22}}\right)x_1^2(k)+\left(1+\frac{h_{22}}{h_{11}}\right)x_2^2(k)\right]+$$
$$+h_{11}L_{12}^2\left[\frac{h_{11}}{h_{22}}x_1^2(k)+x_2^2(k)\right]+h_{22}|a_{22}|L_{21}\left[x_1^2(k)+\left(1+\frac{h_{22}}{h_{11}}\right)x_2^2(k)\right]+$$
$$+h_{22}|a_{22}|L_{22}\left[\frac{h_{11}}{h_{22}}x_1^2(k)+2x_2^2(k)\right]+h_{22}L_{21}^2\left[x_1^2(k)+\frac{h_{22}}{h_{11}}x_2^2(k)\right]+$$
$$+h_{22}L_{21}|L_{12}\left[\left(1+\frac{h_{11}}{h_{22}}\right)x_1^2(k)+\left(1+\frac{h_{22}}{h_{11}}\right)x_2^2(k)\right]+h_{22}L_{22}^2\left[\frac{h_{11}}{h_{22}}x_1^2(k)+x_2^2(k)\right]$$

or

$$\Delta V(x_1(k), x_2(k))$$
$$\leq -[h_{11}(1-a_{11}^2)-h_{11}L_{11}-h_{22}L_{12}]x_1^2(k)-[h_{22}(1-a_{22}^2)-h_{11}L_{21}-h_{22}L_{22}]x_2^2(k),$$

where

$$K_{11}=|a_{11}|\left(2L_{11}+L_{12}\left(1+\frac{h_{11}}{h_{22}}\right)\right)+L_{11}^2+L_{11}L_{12}\left(1+\frac{h_{11}}{h_{22}}\right)+L_{12}^2\frac{h_{11}}{h_{22}},$$
$$K_{12}=|a_{22}|\left(L_{21}+L_{22}\frac{h_{11}}{h_{22}}\right)+L_{21}^2+L_{21}L_{22}\left(1+\frac{h_{11}}{h_{22}}\right)+L_{22}^2\frac{h_{11}}{h_{22}},$$
$$K_{21}=|a_{11}|\left(L_{11}\frac{h_{22}}{h_{11}}+L_{12}\right)+L_{11}^2\frac{h_{22}}{h_{11}}+L_{11}L_{12}\left(1+\frac{h_{22}}{h_{11}}\right)+L_{12}^2,$$
$$K_{22}=|a_{22}|\left(L_{21}\left(1+\frac{h_{22}}{h_{11}}\right)+2L_{22}\right)+L_{21}^2\frac{h_{22}}{h_{11}}+L_{21}L_{22}\left(1+\frac{h_{22}}{h_{11}}\right)+L_{22}^2$$

Finally, conditions of asymptotic stability are

$$h_{11}(1-a11^2)-h_{11}K_{11}-h_{22}K_{12}>0, h_{22}(1-a22^2)-h_{11}K_{21}-h_{22}K_{22}>0,$$

i.e. the inequalities (3.3) hold.

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RADON AND ITS PROGENY IN THE ENVIRONMENT

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Abstract: *In 1900, Dorn discovered the emanation in the uranium series that eventually became the well-known gas ^{222}Rn. But the effects of prolonged exposure had been suspected and noted 300 years earlier among underground miners who developed lung cancer. In 1951, researchers at the university of Rochester N.Y. pointed out that the lung cancer health hazard was from the alpha radiation dose delivered by the radon decay products that deposited in the respiratory tract. The aim of our contribution is measuring of radon concentration in some places at Moravia by means of the Level Living Monitor LLM 500,outstanding equipment not only from the scientific but also from didactical point of view.*

Keywords: radon, daughter products, lung carcinoma, Moravia, LLM 500

INTRODUCTION

Although radon was discovered at the beginning of 20th century, its effects have been known since 16th century. At Schneeberg in the Erz Mountains Agricola noted in 1597 a high frequency of fatal lung diseases occurred among local miners. Similar effects were seen in miners at Jáchymov as early as the 17th century. In these mines were copper, iron and silver ores accompanied by pitchblende – source of uranium and radium.

In 1879 two German physicians, Hartung and Hesle, pointed out that most of the Schneeberg mine deaths were lung cancers. The Schneeberg miners who had worked in the mines for more than ten years developed the Erz Mountain disease, called *bergkrankheit*, or *black death*. Here is the short historical chronology of radon [1]:

1597 Agricola noted high level of what turned out to be lung cancer among Erz Mountain miners.
1896 Becquerel discovers radioactivity of uranium
1898 The Curies and Schmidt discover radioactivity of thorium and also elements radium and polonium
1898 Rutherford discovers alpha and beta particles
1899 Thomson and Rutherford demonstrate that radioactivity causes ionization
1900 Dorn discovers the emanation in the ^{238}U series, which is now called radon
1901 Rutherford and Brooks demonstrate that radon is a radioactive gas
1901 Discovery of active deposit of thorium by Rutherford and of radium by the Curies
1902 Rutherford and Soddy discover transmutation
1902 Thomson discovers radon in tap water
1903 Rutherford and Soddy develop equation describing radioactive decay
1904 Geisel and Debierne discover action
1913 Arnstein identifies squamous-cell carcinoma in autopsy of miner

1913 Fajans discovers group displacement laws
1914 First medical use of radon
1925 First mention of the name radon in the literature
1940s Causal link shown between radon and lung cancer
1941 National Bureau of Standards advisory committee adopts an air radon standard (370 $Bq.m^{-3}$)
1955 Concept of a working level (WL) first suggested
1957 Development of the Lucas cell for detection of radon

1. PHYSICAL PROPERTIES AND SOURCES OF RADON

Radon is a naturally occurring, colorless, odorless, almost chemically inert and radioactive gas. It is the heaviest and has the highest melting point, boiling point, critical temperature and critical pressure from the other noble gasses. Radon is part of the naturally occurring radioactive decay chain from uranium or thorium to stable lead [2] (Fig.1) . Radon decays with emission of α-particle to ^{218}Po which has a half-life approximately 3 minutes and because it is not electrically neutral, adheres to dust. The dust is then inhaled into lungs, where cell-damaging α radiation can occur when ^{218}Po decays. Similar situation repeats by decay of ^{218}Po to radioactive ^{214}Pb and then to radioactive ^{214}Bi.
Because radon is a radioactive, noble gas with no chemical reactivity, its concentration at any point of measurement is a function of three primary factors:

- concentration and distribution of its parent in the source material,
- efficiency of transport processes which bring it into the biosphere,
- its half-life.

Radon is a short-lived member of the ^{238}U decay series and a progeny ^{226}Ra, its concentration is a function of the levels of these elements in the source material. Because one of the most basic properties of uranium and thorium is the tendency to be enriched in rock which have a low melting point, their content is higher in granite than diorites, basalt or limestone. In the Fig.2 there is radon situation in Czech Republic shown.
Radon gas enters homes in three main ways (Fig.3):

1. It migrates up from soil and rocks into basements (cellars) and lower floors of houses.
2. Dissolved in groundwater, it is pumped into wells and then into homes.
3. Radon – contaminated material, such as building blocks, are used in the construction of houses.

The simplest way how radon can be reduced in our houses and other buildings is to locate the entry points of radon and seal them. This action however, is often not sufficient, so additional ventilation to the home, using fans and other device may be necessary. Increased ventilation is the primary remedy for radon problems. If these methods are not successful, a venting system may be constructed [3,4].

2. HEALTH EFFECTS

Once inhaled, radon gas quickly finds its way to the blood stream. It is a chemically inert gas and only a small fraction of that inhaled will be absorbed by the blood and not exhaled. Further, because the half – life of radon is relatively long compared to breathing time, only a small amount of it will decay while in the lung. Acute and sub-acute early effects, as well as late effects, can be expected following exposure of the respiratory tract to radon progeny. High concentrations of radon decay products in the lungs of animals can result in profound structural and functional changes that may produce lifespan – shortening, pulmonary emphysema, pulmonary fibrosis and lung cancer. Many of the more than 40 distinctive cell types of the respiratory tract could be affected [1,5]. The nature and magnitude of biological effects that may occur following inhalation of radon decay products will depend on many factors, such as fractions deposited in the respiratory tract and their retention times, translocation to other tissues and rate of excretion to the body.

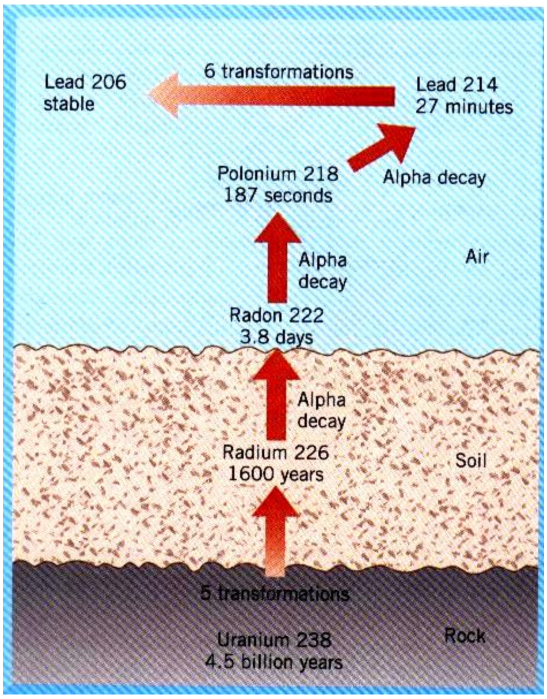

Fig.1. Simplified diagram of radioactive decay chain for radon [4]

Inhaled short-lived radon decay product will, to a large extent, decay at their deposition site. Consequently, the tissues in the nasopharynx, the tracheobronchial tree and the pulmonary region receive the majority of the radiation dose. The dose to the bronchi generally predominates in humans. These sites contain precursor or stem cells that are particularly sensitive to the cytotoxic and carcinogenic properties of α-emitting radon progeny. They may be more sensitive to carcinogenesis because of exposure to other environmental agents (such as cigarette smoke) that may increase cell division [5].

3. EXPERIMENTAL RESULTS

The concentration of radon and its daughter products in many parts of Moravia has been measured. For this purpose we used outstanding equipment Level Living Monitor LLM 500 (Műnchener Apparatebau fűr elektronische Geräte GmbH, Germany) [6,7,8]. The monitor consists of a dealer large area proportional detector. The efficiency is enhanced by using a β-reflector. A mechanical code inside the filter mouth recognizes the correct insertion of the loaded side of the filter diskette. The reliable portable sampler consists of a powerful turbine with a precise readout. The sinter diskette with very low flow resistance supports the filter material and reduces the noise level.
All measurements have been made in all parts of Moravia from the years 1997-2010 predominantly in unventilated cellars. In the Fig 4. there are shown as an example the results

of a typical measurements (South – Western Moravia, the Javořice Mountains, beginning of June 2007). The numbers in the figure mean the highest and the lowest value of radon concentration in the measured place. Because the concentration of radon is time and place dependent, the error has not been counted.

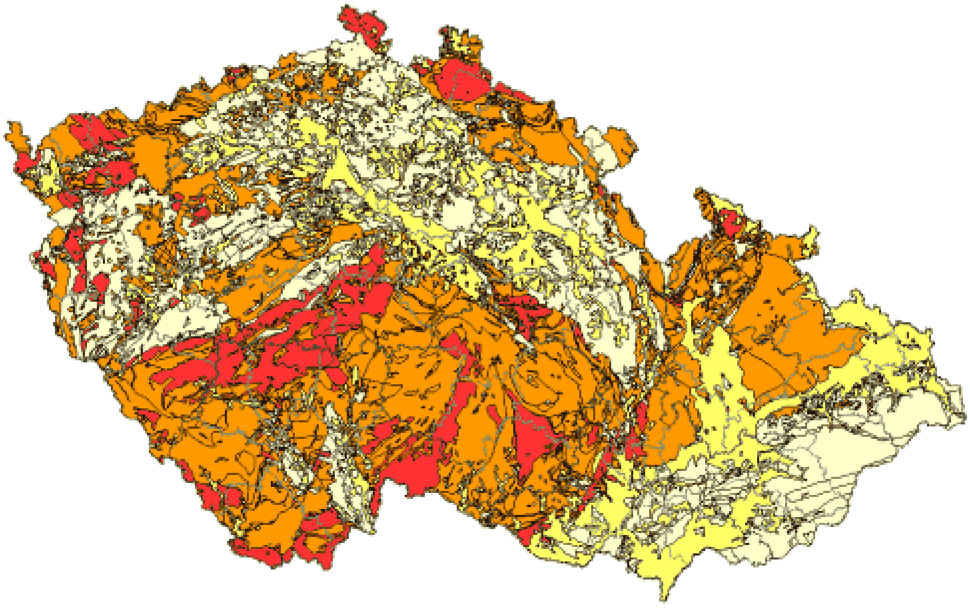

Fig.2. Radon situation in Czech Republic [9]

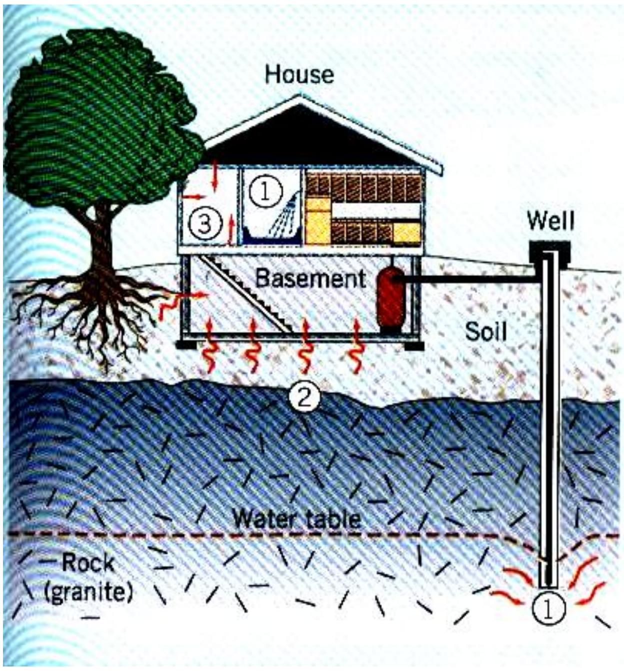

Fig.3. How radon may enter homes [4]

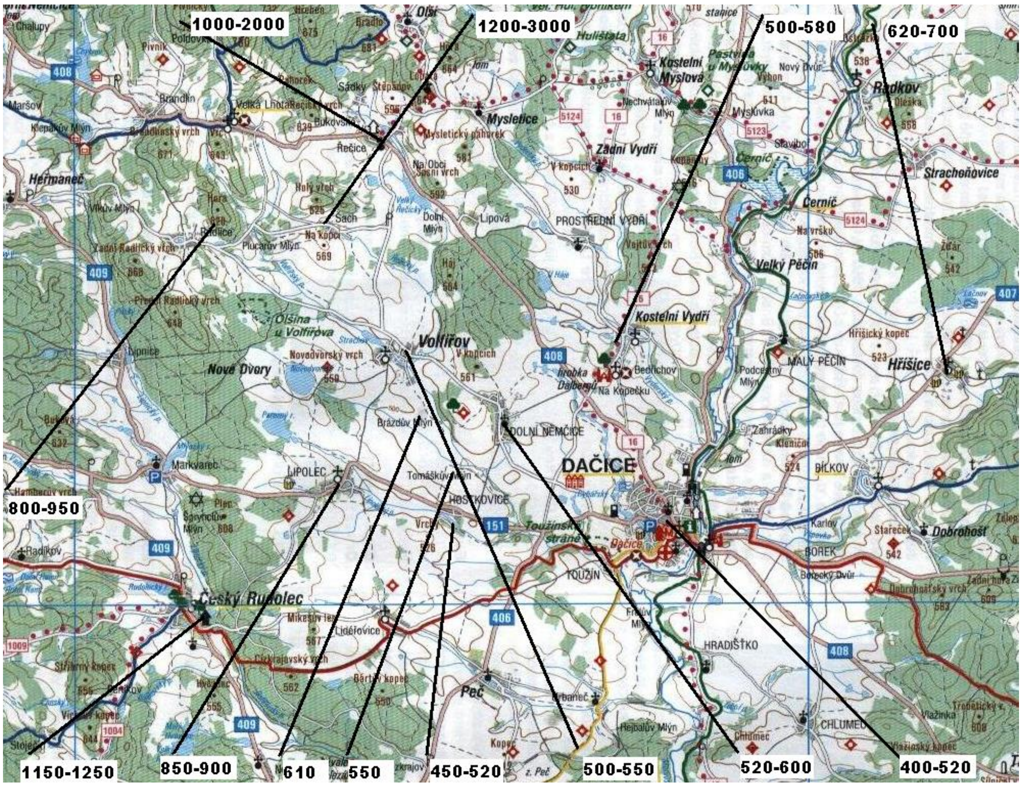

Fig.4. Measurements of radon concentration (Southwestern Moravia, June 2007)

Tab.1.

City, Village	No	Min – Max Activity Bq.m^{-3}	City, Village	No	Min – Max Activity Bq.m^{-3}
Dolní Rožínka	3	800 - 1900	Dačice	5	400 - 520
Znojmo	4	140 - 360	Dolní Němčice	8	320 - 460
Velké Meziříčí	2	220 - 380	Radlice	6	600 - 920
Jihlava	4	120 - 420	Šach	3	1100 - 1300
Ostrava	4	80 - 110	Volfířov	6	480 - 560
Uh. Hradiště	4	30 - 100	Kostelní Vydří	4	220 - 410
Hodonín	3	60 - 110	Mor. Budějovice	2	80 - 120
Bučovice	2	40 - 80	Jemnice	2	60 - 130
Hrotovice	3	60 - 180	Třebíč	4	360 - 450
Telč	6	360 - 480	Brno-Bystrc	8	90 - 210

From the figure we can see increasing of activity in left part of the map (granite bedrock) in comparison with right part of the map (gneiss bedrock). The highest concentration of radon was indicated on the boundary between granite and gneiss bedrocks-geological break . As we knew, in the break the deposits of uranium were found.
In the Tab.1. there are the results of our measurements of radon concentration, measured in unventilated cellars at various parts of Moravia (No is the number of cellars in the city, or village).

CONCLUSION

The radiation dose from inhaled decay product of ^{222}Rn is the dominant component of natural radiation exposures of the general population. The average lifetime risk of lung cancer caused by exposure to radon decay products is estimated to be more than 400 causes of lung cancer annually among Czech population. This average risk is more than that received on the average from all other natural radiation sources or from medical exposures.
Very important source of radon there is also domestic water drawn from underground sources. Surface waters have radon concentration too small to affect indoor levels when used domestically, but ground water is a good position to accumulate radon generated within the earth´s crust. Concentration of daughter products both in the air and in the ground water has been measured at the Department of Physics, Chemistry and Vocational Education, Faculty of Education, Masaryk University Brno, Czech Republic, by LLM-500 [6,8,10]. The results of the measurements carried out by means of said apparatus are shown in Fig.4 and in Tab.1. Measurements are to be understood as an immediate values, as the concentration of radon in soil and water varies mainly depending on the season (winter is higher).

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SUMS OF GENERALIZED ALTERNATING HARMONIC SERIES WITH PERIODICALLY REPEATED NUMERATORS $(\mathbf{1},\boldsymbol{a})$ AND $(\mathbf{1},\mathbf{1},\boldsymbol{a})$

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Abstract: *This paper deals with certain generalization of the alternating harmonic series – the generalized harmonic series with periodically repeated numerators* $(1, a)$ *and* $(1,1,a)$*. Firstly, we show that the only one value of the numerator* a *of the first series, for which the series converges, is* $a = -1$ *and that its sum is* $\ln 2$*. Further, we derive that the only one value of the numerator* a *of the second series, for which the series converges, is* $a = -2$ *and that its sum is* $\ln 3$*. Finally, we verify this analytically obtained result and compute the sum of this series by using the computer algebra system Maple 15 and its basic programming language.*

Keywords: harmonic series, alternating harmonic series, geometric series, sum of the series.

INTRODUCTION AND BASIC NOTIONS

Let us recall the basic terms and notions. The *harmonic series* is the sum of reciprocals of all natural numbers except zero (see e.g. web page [3]), so this is the series

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \cdots .$$

The divergence of this series can be easily proved e.g. by using the integral test or the comparison test of convergence.

The series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots$$

is known as the *alternating harmonic series*. This series converges by the alternating series test. In particular, the sum (interesting information about sum of series can be found e.g. in book [1] or paper [2]) is equal to the natural logarithm of 2:

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots = \ln 2.$$

This formula is a special case of the *Mercator series*, the Taylor series for the natural logarithm:

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots .$$

The series converges to the natural logarithm (shifted by 1) whenever $-1 < x \leq 1$.

1. SUM OF GENERALIZED ALTERNATING HARMONIC SERIES WITH PERIODICALLY REPEATED NUMERATORS $(\mathbf{1}, \boldsymbol{a})$

We deal with the numerical series of the form

$$\sum_{n=1}^{\infty}\left(\frac{1}{2n-1}+\frac{a}{2n}\right)=\frac{1}{1}+\frac{a}{2}+\frac{1}{3}+\frac{a}{4}+\frac{1}{5}+\frac{a}{6}+\frac{1}{7}+\frac{a}{8}+\cdots, \tag{1}$$

where obviously $a<0$. This series we shall call *generalized convergent harmonic series with periodically repeated numerators* $(1,a)$. Our aim is to determine the values of the numerator a, for which the series (1) converges, and the sum of such series.

The power series corresponding to the series (1) has evidently the form

$$\sum_{n=1}^{\infty}\left(\frac{x^{2n-1}}{2n-1}+\frac{ax^{2n}}{2n}\right)=\frac{x}{1}+\frac{ax^2}{2}+\frac{x^3}{3}+\frac{ax^4}{4}+\frac{x^5}{5}+\frac{ax^6}{6}+\frac{x^7}{7}+\frac{ax^8}{8}+\cdots. \tag{2}$$

We denote its sum by $s(x)$. The series (2) is for $x\in(-1,1)$ absolutely convergent, so we can rearrange it (one interesting paper about rearranging the series is [4]) and rewrite it in the form

$$s(x)=\sum_{n=1}^{\infty}\frac{x^{2n-1}}{2n-1}+a\sum_{n=1}^{\infty}\frac{x^{2n}}{2n}=\frac{x}{1}+\frac{x^3}{3}+\frac{x^5}{5}+\cdots+a\left(\frac{x^2}{2}+\frac{x^4}{4}+\frac{x^6}{6}+\cdots\right). \tag{3}$$

If we differentiate the series (3) term-by-term, where $x\in(-1,1)$, we get

$$s'(x)=\sum_{n=1}^{\infty}x^{2n-2}+a\sum_{n=1}^{\infty}x^{2n-1}=(1+x^2+x^4+\cdots)+a(x+x^3+x^5+\cdots). \tag{4}$$

After reindexing and fine arrangement the series (4) for $x\in(-1,1)$ we obtain

$$s'(x)=\sum_{n=0}^{\infty}x^{2n}+ax\sum_{n=0}^{\infty}x^{2n},$$

that is

$$s'(x)=(1+ax)\sum_{n=0}^{\infty}(x^2)^n. \tag{5}$$

When we summate the series (5), which represent a convergent geometric series with the first term 1 and with the ratio x^2, where $|x^2|<1$, i.e. for $x\in(-1,1)$, we get

$$s'(x)=(1+ax)\frac{1}{1-x^2}.$$

Be means of the CAS Maple 15 we convert this fraction to partial fractions and get

$$s'(x)=\frac{1+a}{2(1-x)}+\frac{1-a}{2(1+x)},$$

where $x\in(-1,1)$. The sum $s(x)$ of the series (2) we obtain by integration in the form

$$s(x)=\int\left(\frac{1+a}{2(1-x)}+\frac{1-a}{2(1+x)}\right)\mathrm{d}x=-\frac{1+a}{2}\ln(1-x)+\frac{1-a}{2}\ln(1+x)+C.$$

From the condition $s(0)=0$ we obtain $0=C$, hence

$$s(x) = \frac{1-a}{2}\ln(1+x) - \frac{1+a}{2}\ln(1-x)\,. \tag{6}$$

Now, we will deal with the convergence of the series (2) in the right point $x = 1$. After substitution $x = 1$ to the power series (2) – it can be done by the extended version of Abel's theorem (see e.g. [5], p. 23) – we get the numerical series (1). By the integral test we can prove that the series (1) converges if and only if $a = -1$. Its sum by (6) then equals

$$s(1) = \ln 2 \doteq 0.693147.$$

2. SUM OF GENERALIZED ALTERNATING HARMONIC SERIES WITH PERIODICALLY REPEATED NUMERATORS $(\mathbf{1}, \mathbf{1}, \boldsymbol{a})$

Now, we deal with the numerical series of the form

$$\sum_{n=1}^{\infty}\left(\frac{1}{3n-2} + \frac{1}{3n-1} + \frac{a}{3n}\right) =$$

$$= \frac{1}{1} + \frac{1}{2} + \frac{a}{3} + \frac{1}{4} + \frac{1}{5} + \frac{a}{6} + \frac{1}{7} + \frac{1}{8} + \frac{a}{9} + \frac{1}{10} + \frac{1}{11} + \frac{a}{12} + \cdots, \tag{7}$$

where obviously $a < 0$. This series we shall call *generalized convergent harmonic series with periodically repeated numerators* $(1,1,a)$. We determine the values of the numerator a, for which the series (7) converges, and the sum of this series.

The power series corresponding to the series (7) has evidently the form

$$\sum_{n=1}^{\infty}\left(\frac{x^{3n-2}}{3n-2} + \frac{x^{3n-1}}{3n-1} + \frac{ax^{3n}}{3n}\right) =$$

$$= \frac{x}{1} + \frac{x^2}{2} + \frac{ax^3}{3} + \frac{x^4}{4} + \frac{x^5}{5} + \frac{ax^6}{6} + \frac{x^7}{7} + \frac{x^8}{8} + \frac{ax^9}{9} + \frac{x^{10}}{10} + \frac{x^{11}}{11} + \frac{ax^{12}}{12} + \cdots. \tag{8}$$

We denote its sum by $t(x)$. The series (8) is for $x \in (-1,1)$ absolutely convergent, so we can rearrange it and rewrite it in the form

$$t(x) = \sum_{n=1}^{\infty}\frac{x^{3n-2}}{3n-2} + \sum_{n=1}^{\infty}\frac{x^{3n-1}}{3n-1} + a\sum_{n=1}^{\infty}\frac{x^{3n}}{3n}\,. \tag{9}$$

If we differentiate the series (9) term-by-term, where $x \in (-1,1)$, we get

$$t'(x) = \sum_{n=1}^{\infty} x^{3n-3} + \sum_{n=1}^{\infty} x^{3n-2} + a\sum_{n=1}^{\infty} x^{3n-1}\,. \tag{10}$$

After reindexing and fine arrangement the series (10) for $x \in (-1,1)$ we obtain

$$t'(x) = \sum_{n=0}^{\infty} x^{3n} + x\sum_{n=0}^{\infty} x^{3n} + ax^2\sum_{n=0}^{\infty} x^{3n}\,,$$

that is

$$t'(x) = (1 + x + ax^2)\sum_{n=0}^{\infty}(x^3)^n\,. \tag{11}$$

When we summate the convergent geometric series (11) which has the first term 1 and the ratio x^3, where $|x^3| < 1$, i.e. for $x \in (-1,1)$, we get

$$t'(x) = (1 + x + ax^2)\frac{1}{1 - x^3}.$$

We convert this fraction using the CAS Maple 15 to partial fractions and get

$$t'(x) = \frac{2 + a}{3(1 - x)} + \frac{(1 - a)(2x + 1)}{3(x^2 + x + 1)},$$

where $x \in (-1,1)$. The sum $t(x)$ of the series (8) we obtain by integration in the form

$$t(x) = \int \left(\frac{2 + a}{3(1 - x)} + \frac{(1 - a)(2x + 1)}{3(x^2 + x + 1)}\right) \mathrm{d}x =$$

$$= -\frac{2 + a}{3}\ln(1 - x) + \frac{1 - a}{3}\ln(x^2 + x + 1) + C.$$

From the condition $t(0) = 0$ we obtain $0 = C$, hence

$$t(x) = -\frac{2 + a}{3}\ln(1 - x) + \frac{1 - a}{3}\ln(x^2 + x + 1) + C. \tag{12}$$

Now, we will deal with the convergence of the series (8) in the right point $x = 1$. After substitution $x = 1$ to the power series (8) – it can be done by the extended version of Abel's theorem (see [5], p. 23) – we get the numerical series (7). By the integral test we can prove that the series (7) converges if and only if $a = -2$. Its sum by (12) then equals

$$t(1) = \ln 3 \doteq 1.098612.$$

3. NUMERICAL SOLUTION

Besides the problem to determine the sum of the numerical series (1), where $a = -1$ and which give the well-known result $s = \ln 2$, we solved the problem to determine the sum of the series (7), where $a = -2$ and with the sum $t = \ln 3$, by using the basic programming language in the computer algebra system Maple 15. It was used the following procedure:

```
sumgenhar11-2:=proc(n,p)
    local r,i,k,t;
    s:=0;
    r:=0;
    for i from 0 to n-1 do
          for k from p*i+1 to p*(i+1) do
                r:=1/(3*k-2)+1/(3*k-1)-2/(3*k);
                t:=t+r;
          end do;
          print("n=",k-1,"t=",evalf[20](t));
    end do;
end proc:
```

```
sumgenhar11-2(1,1000);
sumgenhar11-2(9,10000);
sumgenhar11-2(10,100000);
```

The results of this procedure can be written into a table below, where the numbers of triplets

$$\left(\frac{1}{3n-2}+\frac{1}{3n-1}-\frac{2}{3n}\right)$$

are stated in the first and in the third columns:

# of triplets	sum t	# of triplets	sum t
1000	1.098279029	100000	1.098608955
10000	1.098578956	200000	1.098610622
20000	1.098595622	300000	1.098611178
30000	1.098601178	400000	1.098611455
40000	1.098603956	500000	1.098611622
50000	1.098605622	600000	1.098611733
60000	1.098606733	700000	1.098611812
70000	1.098607527	800000	1.098611872
80000	1.098608122	900000	1.098611918
90000	1.098608585	1000000	1.098611955

Fig. 1. The table with the approximate values of the sum t of the generalized harmonic series with periodically repeating numerators $(1,1,-2)$

Computation of these 20 values took about 1670 seconds, i.e. almost 28 minutes. At another computation which took about 48420 seconds, i.e. 13 hours 27 minutes, the following three values were determined: for 2000000 triplets $(1,1,-2)$ of the numerators the sum t is approximately 1.098612122, for 3000000 triplets the sum t is about 1.098612178, and for 4000000 triplets is $t \doteq 1.098612205$, while $\ln 3 = 1.098612288\ldots$, so the difference between the last computed sum and its exact value is only about $8 \cdot 10^{-8}$.

CONCLUSION

This paper deals with certain generalization of the alternating harmonic series – the generalized harmonic series with periodically repeated numerators $(1,a)$ and $(1,1,a)$ and with determination the sums of these two types of numerical series. Firstly, we show that the only one value of the numerator a of the first series (1), for which the series converges, is $a=-1$ and analytically we derive that its sum is $\ln 2$, i.e. we get the well-known result

$$\sum_{n=1}^{\infty}\left(\frac{1}{2n-1}-\frac{1}{2n}\right)=\frac{1}{1}-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}-\frac{1}{8}+\cdots=\ln 2\,.$$

Further, we derive that the only one value of the numerator a of the second series (7), for which the series converges, is $a=-2$ and its sum is $\ln 3$, i.e.

$$\sum_{n=1}^{\infty}\left(\frac{1}{3n-2}+\frac{1}{3n-1}-\frac{2}{3n}\right)=\frac{1}{1}+\frac{1}{2}-\frac{2}{3}+\frac{1}{4}+\frac{1}{5}-\frac{2}{6}+\frac{1}{7}+\frac{1}{8}-\frac{2}{9}+\cdots=\ln 3\,.$$

Finally, we verify this result by computing this sum by using the computer algebra system Maple 15 and its basic programming language. It can be said that these two generalized alternating harmonic series belong to special types of convergent infinite series, such as geometric and telescoping series, which sum can be found analytically and also presented by means of a simple numerical expression.

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APPLICATIONS OF MATHEMATICS IN TECHNICAL MECHANICS USING WOLFRAM|ALPHA

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Abstract: *The paper combines the use of mathematics together with the application of the computational tool Wolfram|Alpha in technical mechanics. First moments of area, centroids and moments of inertia with respect to centroidal axes are derived for some standard cross-sections using knowledge of integral calculus of one variable only. The relationship of moments of inertia with respect to parallel axes is stated and Steiner's theorem is applied. Mathematical computations are illustrated through Wolfram|Alpha.*

Keywords: mechanics, mathematics, computer algebra system, first moment of area, moment of inertia, centroid

INTRODUCTION

In technical mechanics and mechanics of materials, formulas are used for the calculations of cross-sectional areas, first moments of an area, coordinates of a centroid and moments of inertia of common geometrical shapes. These formulas have already been derived and are presented in tables, some of them are listed in Tab. 1. Other cross-sectional characteristics can be calculated using them for example a radius of gyration and a section modulus.
Within the innovation of the study programs at Faculty of Forestry and Wood Technology at Mendel University in Brno, the mentioned formulas in subject Technical mechanics have been derived for the standard planar cross-sections: a square, a rectangle, a right triangle, an isosceles triangle, a quarter circle, a half circle, a circle, a hollow rectangle and an annulus, see [5]. In this paper, we show computations of some of them. First moments of an area and moments of inertia of planar cross-sections are usually evaluated by double integrals that are not discussed in the basic course of mathematics. As differential elements of an area are commonly used in engineering subjects, the knowledge of integral calculus of one real variable is applied in these computations only. Steiner's theorem is used for the derivation of moments of inertia. It gives relationship between the moment of inertia about an axis passing through a centroid and the moment of inertia about any parallel axis.
The quoted calculations are completed by computational tool Wolfram|Alpha.

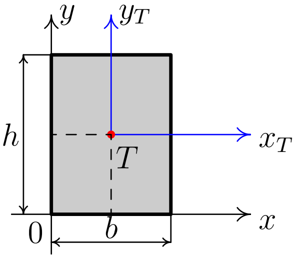
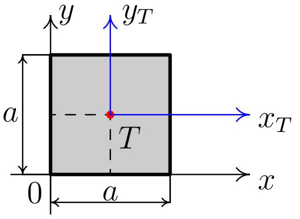
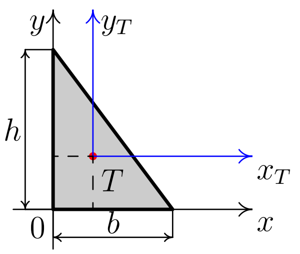
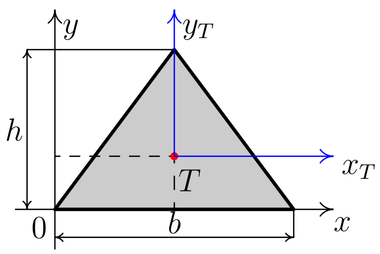
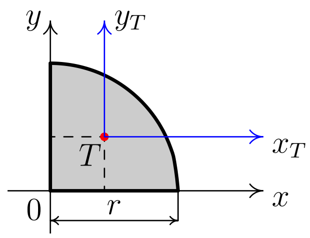
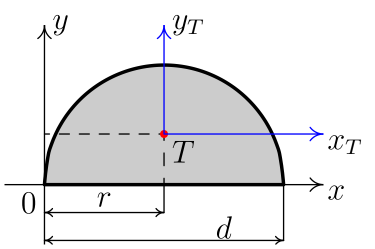
1. COMPUTATION OF FIRST MOMENTS OF AN AREA AND CENTROID

We use a notation common in technical mechanics. Let us remind that an area of a planar cross-section A is defined

$$A = \int_A \mathrm{d}A.$$

First moments of an area about the axes x and y are given by formulas

$$S_x = \int_A y \,\mathrm{d}A, \quad S_y = \int_A x \,\mathrm{d}A. \tag{1}$$

Figure	Area	Centroid	Moment of inertia
	$A = bh$	$x_T = \frac{1}{2}b$ $y_T = \frac{1}{2}h$	$I_{x_T} = \frac{1}{12}bh^3$ $I_{y_T} = \frac{1}{12}hb^3$
	$A = a^2$	$x_T = \frac{1}{2}a$ $y_T = \frac{1}{2}a$	$I_{x_T} = \frac{1}{12}a^4$ $I_{y_T} = \frac{1}{12}a^4$
	$A = \frac{1}{2}bh$	$x_T = \frac{1}{3}b$ $y_T = \frac{1}{3}h$	$I_{x_T} = \frac{1}{36}bh^3$ $I_{y_T} = \frac{1}{36}hb^3$
	$A = \frac{1}{2}bh$	$x_T = \frac{1}{2}b$ $y_T = \frac{1}{3}h$	$I_{x_T} = \frac{1}{36}bh^3$ $I_{y_T} = \frac{1}{48}hb^3$
	$A = \frac{1}{4}\pi r^2$	$x_T = \frac{4r}{3\pi}$ $y_T = \frac{4r}{3\pi}$	$I_{x_T} = r^4\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)$ $I_{y_T} = r^4\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)$
	$A = \frac{1}{2}\pi r^2$	$x_T = r$ $y_T = \frac{4r}{3\pi}$	$I_{x_T} = r^4\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$ $I_{y_T} = \frac{1}{8}\pi r^4$

Tab. 1. Moments of inertia of some standard shapes

Coordinates of a centroid of an area are determined by

$$x_T = \frac{S_y}{A}, \quad y_T = \frac{S_x}{A}. \tag{2}$$

If the cross-section has one axis of symmetry, the centroid lies on it. In the case two or more axes of symmetry the centroid is located at the intersection of these axes.
To compute first moments of a planar cross-section we do not apply the standard procedure using double integral, but we start from a physical approach commonly used in engineering as in [1] or [2]. The given cross-section is divided into an infinite number of infinitely narrow strips. If we derive the first moment S_x, the strips are parallel to the axis x, for the first moment S_y they are parallel to the axis y.
We show the computation procedure of the first moment S_y of a quarter circle by which x coordinate of the centroid will be calculated.

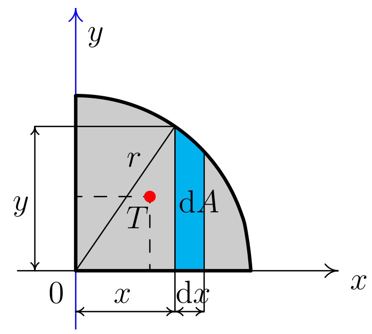


Fig. 1. Quarter circle cross-section

The given quarter circle of a radius r, sketched in Fig. 1., is divided into infinitely small strips parallel to the axis y. Every strip has the width $\mathrm{d}x$ and the height y so its area is $\mathrm{d}A = y\,\mathrm{d}x$ where $\mathrm{d}A$ is the proper differential element of an area.
From the equation of a circle $x^2 + y^2 = r^2$ we have $y = \sqrt{r^2 - x^2}$ for $y \geq 0$. Thus

$$\mathrm{d}A = \sqrt{r^2 - x^2}\,\mathrm{d}x.$$

As $0 \leq x \leq r$ holds, substituting the previous relation into the formula 1 we obtain

$$S_y = \int_A x\,\mathrm{d}A = \int_0^r x\sqrt{r^2 - x^2}\,\mathrm{d}x = \left| \begin{array}{c} t = r^2 - x^2 \\ \mathrm{d}t = -2x\,\mathrm{d}x \end{array} \right| = -\frac{1}{2}\int_{r^2}^0 \sqrt{t}\ \mathrm{d}t =$$

$$= \frac{1}{2}\int_0^{r^2} t^{\frac{1}{2}}\,\mathrm{d}t = \frac{1}{3}\left[\sqrt{t^3}\right]_0^{r^2} = \frac{1}{3}r^3.$$

We get the same result by using free online service Wolfram|Alpha at the web address http://wolframalpha.com that allows to perform mathematical calculations. The syntax of this tool is simple you need to know keywords only. A command is specified by the name of the operation in English. More information can be found at http://wolframalpha.com/examples and for example in [3], [6] and [4].
To calculate the first moment S_y of a quarter circle we write the following command into an input field
`integrate x*sqrt(r^2-x^2) dx from 0 to r`.

To determine the centroid coordinate x_T we substitute the previous calculated first moment $S_y = \frac{1}{3}r^3$ and the known area of a quarter circle $A = \frac{1}{4}\pi r^2$ into the relation 2. Then we have

$$x_T = \frac{\frac{1}{3}r^3}{\frac{1}{4}\pi r^2} = \frac{4r}{3\pi}.$$

As Wolfram|Alpha does not allow to store subresults, to calculate the centroid coordinate we have to either write the obtained result $\frac{1}{3}r^3$ into a command or use the written command for the calculation of S_y in the input field and divide it by the area of a quarter circle `(integrate x*sqrt(r^2-x^2) dx from 0 to r)/((1/4)*pi*r^2)`.

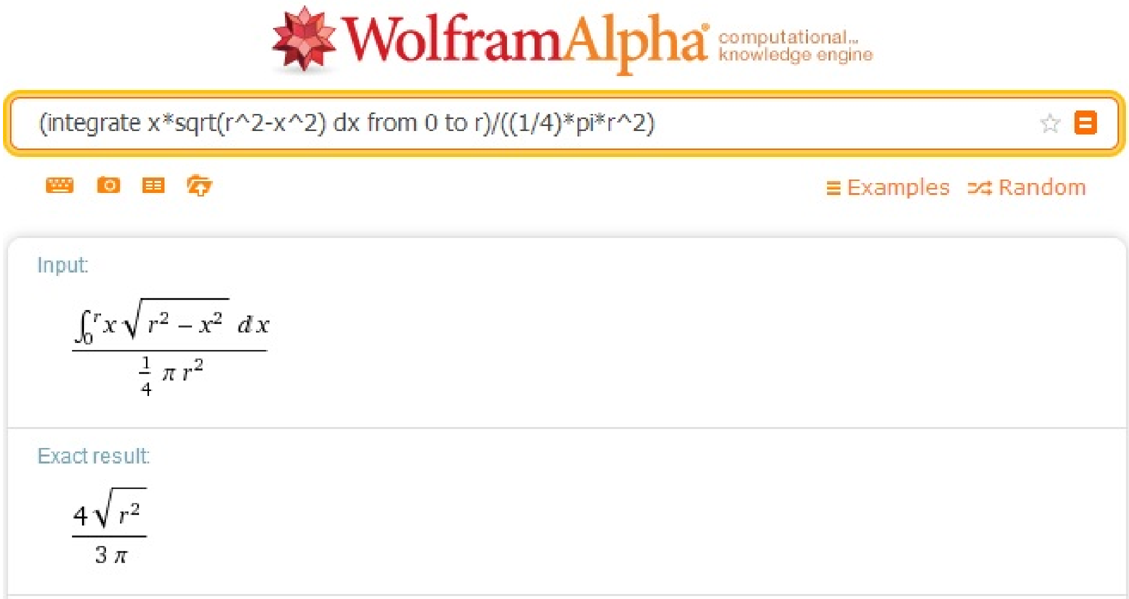


Fig. 2. Centroid coordinate of quarter circle using Wolfram|Alpha

As you can see in Fig. 2., for $r > 0$ we get the same formula of the coordinate x_T.
The first moment S_x and the centroid coordinate y_T of a quarter circle can be derived in a similar way. However, because a quarter circle has only one axis of symmetry, the centroid lies on it and the coordinate y_T is equal to the coordinate x_T, ie. $\frac{4r}{3\pi}$.

2. COMPUTATION OF MOMENTS OF INERTIA

Moments of inertia of a planar cross-section A about the axes x and y are defined by the following formulas

$$I_x = \int_A y^2 \, \mathrm{d}A, \quad I_y = \int_A x^2 \, \mathrm{d}A. \tag{3}$$

In tables, there are listed moments of inertia about centroidal axes. But in engineering, it is often easier to calculate these moments with respect to some other axes and then recalculate them with respect to the axes passing through a centroid. For this simplification Steiner's theorem, also known as Parallel axis theorem, is used. It shows a relationship between moments of inertia with respect to two different parallel axes one of which passes through a centroid:

$$\begin{aligned} I_x &= I_{x_T} + c^2 A, \\ I_y &= I_{y_T} + d^2 A, \end{aligned} \tag{4}$$

where I_{x_T}, I_{y_T} denote moments of inertia of an area A about the centroidal axes and c, d are the distances of the axes parallel to each other.
Now we derive a moment of inertia of a right triangle about the axis x passing through a vertex of the triangle which is simpler. The origin of the axes is located at this vertex. The calculated moment together with Steiner's theorem will be used for the derivation of the moment of inertia about the centroidal axis.

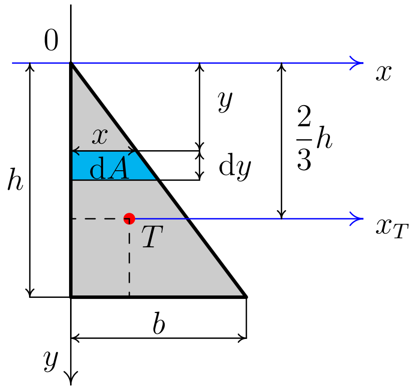


Fig. 3. Right triangle cross-section

Similarly to the first moment, the cross-section is divided into an infinite number of infinitely thin strips parallel to the axis x. The height and the length of the strip are $\mathrm{d}y$ and x respectively and its area is $\mathrm{d}A = x\,\mathrm{d}y$. From similarity of triangles follows

$$\frac{x}{b} = \frac{y}{h} \quad \Rightarrow \quad x = \frac{b}{h}y.$$

We substitute the proper differential element of an area

$$\mathrm{d}A = \frac{b}{h}y\,\mathrm{d}y$$

into the relation 3 and derive

$$I_x = \int_A y^2\,\mathrm{d}A = \int_0^h \frac{b}{h}y^3\,\mathrm{d}y = \frac{b}{h}\left[\frac{y^4}{4}\right]_0^h = \frac{1}{4}bh^3. \tag{5}$$

To determine this moment of inertia through computational engine Wolfram|Alpha we enter the command
`integrate (b/h)*y^3 dy from 0 to h.`
For a conversion of the moment of inertia with respect to the centroidal axis x_T we use relation

$$I_{x_T} = I_x - c^2 A$$

that follows from Steiner's theorem 4. As the centroid of the right triangle in Fig. 3. is at the point $\left(\frac{1}{3}b, \frac{2}{3}h\right)$, we put $c = \frac{2}{3}h$, $I_x = \frac{1}{4}bh^3$ and $A = \frac{1}{2}bh$. Then we get

$$I_{x_T} = \frac{1}{4}bh^3 - \left(\frac{2}{3}h\right)^2 \frac{1}{2}bh = \frac{1}{36}bh^3.$$

With service Wolfram|Alpha we can write the previous relation by using the command for the calculation of definite integral mentioned above
`(integrate (b/h)*y^3 dy from 0 to h)-((2/3*h)^2)*(1/2*b*h)`.

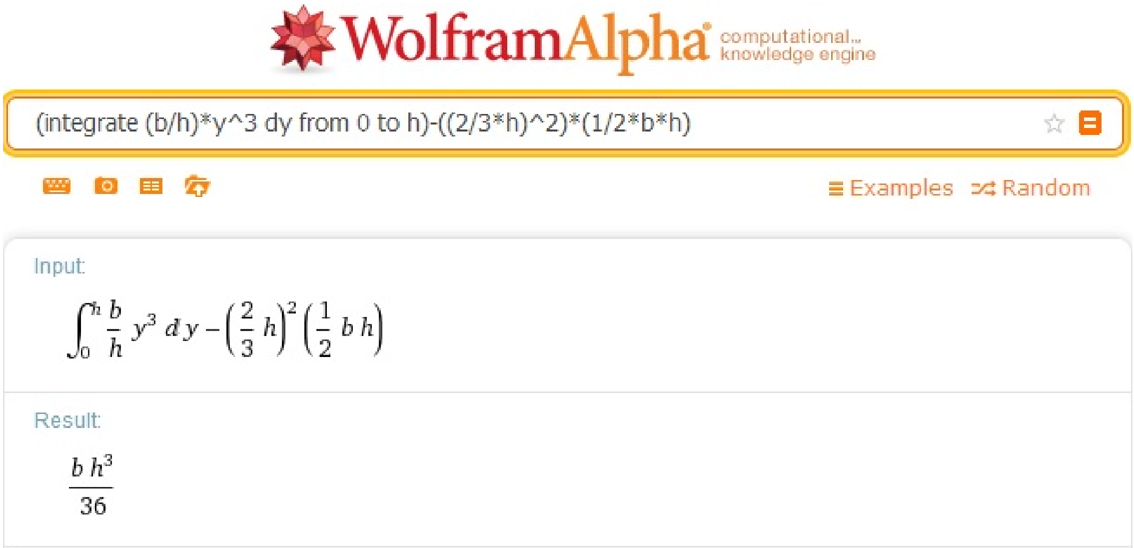


Fig. 4. Moment of inertia using Steiner's theorem and Wolfram|Alpha

The moment of inertia I_{y_T} can be computed in a similar way. In our considerations, we only swap the sides b and h. Thus $I_{y_T} = \dfrac{1}{36} h b^3$.

FINAL REMARKS

By analogy, first moments, coordinates of a centroid and moments of inertia can be derived for all planar cross-sections mentioned in the introduction, see [5]. For an isosceles triangle, a half circle and a circle the division into two simpler shapes according to an axis of symmetry is used - two right triangles, two quarter circles and two semicircles respectively. Moments of inertia of the whole cross-section with respect to the given axes are equal to the sum of moments of its parts with respect to the same axes. In this case, they are equal to the double of moment of inertia of its symmetric part.
Wolfram|Alpha also allows to compute first moments of an area and moments of inertia by double integrals. For example, the moment of inertia 5 of a right triangle can be calculated using the following command
`integrate y^2 dx dy, y=0 to h, x=0 to b/h*y`.

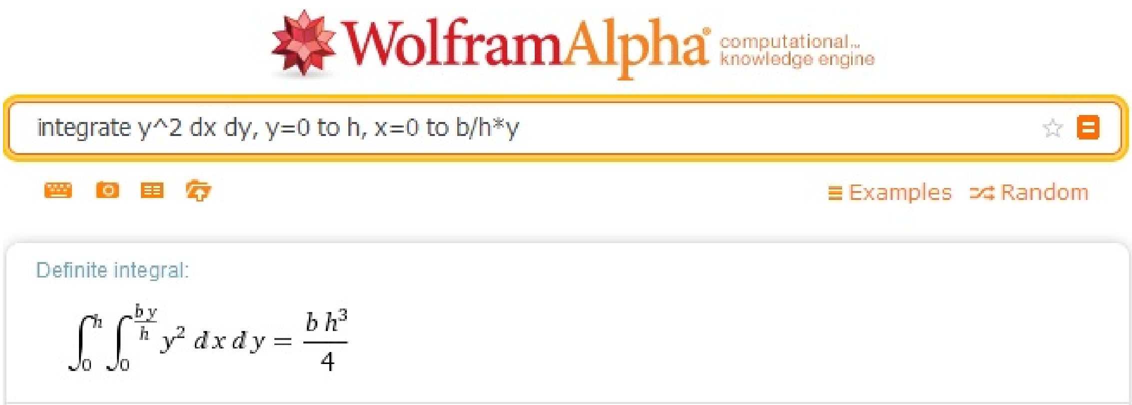


Fig. 5. Moment of inertia using double integral and Wolfram|Alpha

CONCLUSION

The main aim of this paper is to show the connection between mathematics and mechanics as well as the simultaneous application of computational engine Wolfram|Alpha. The presented examples illustrate the use of the acquired mathematical knowledge in mechanics. The computation of the table formulas serve to better and deeper understanding of the subject matter. In this way, students can check that the formulas used in mechanics are valid and mathematical procedure gives the same result as the computational tool. The simultaneous application of Wolfram|Alpha variegates and improves teaching and increases the interest of students. Moreover, the use of this computational service is simple and free available.
Finally, the paper emphasizes the need for mathematics as well as the ability to apply an appropriate computer algebra system in technical subjects. The intention was to improve the motivation of students and teachers themselves and to streamline the technical education.
In addition to technical universities with an agricultural specialization, this approach can be also applied in teaching of mechanics in secondary schools in the field of engineering.

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OPTIMIZATION METHOD OF ABSOLUTE STABILITY CONDITIONS CONSTRUCTING FOR NONLINEAR DIRECT CONTROL SYSTEMS

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Abstract: *At research of the dynamics of different kind systems often used the direct method of Lyapunov. The main theorems of Lyapunov stability and asymptotic stability have necessary and sufficient conditions. This means that if the zero solution of the system is asymptotically stable, then the Lyapunov function always exists. But the central and unresolved problem is actual construction of this function. The problem is simplified if the function is sought in advance parametrically given class of functions, such as the class of quadratic forms. Or in the case of absolute stability problems - in a class of quadratic form plus integral from the nonlinearity, and others, so to say, derived from this type form. In this case, the task of finding the "best" Lyapunov function can be reduced to a convex programming problem.*

Keywords: absolute stability, Lyapunov method, convex optimization problem, optimal Lyapunov function and functional

INTRODUCTION

We will consider absolute stability problem of nonlinear Lur'e control systems in traditional formulation [1-3]. All main definition from these books will be used at present article. A historical overview of the absolute stability problem can be found in [4] or in the introduction of [5]. We will used the next notation $\lambda_{\max}(\cdot)$, $\lambda_{\min}(\cdot)-$ extremal eigenvalue of correspondent symmetric matrices, $H \geq 0$ - positive semi-definiteness of matrix H, $|H| = \lambda_{\max}(H)$ - matrix norm, $|(H, \beta, \nu)| = \sqrt{|H|^2 + \beta^2 + \nu^2}$ - Euclidean norm in space $((R^n \times R^n), R^1, R^1)$, $\langle \cdot, \cdot \rangle$ - scalar product of correspondent vectors. The article consists of introduction, main part and conclusion.

1. MAIN RESULT

Let is consider Lur'e type direct control system of next type

$$\dot{x}(t) = Ax(t) + bf(\sigma(x(t))), \quad \sigma(x(t)) = c^T x(t). \tag{1}$$

Here $b, c, x(t) \in R^n$, $A-$ $n \times n$ asymtotic stable matrix. Nonlinear scalar function $f(\sigma)$ satisfy next linear restriction

$$0 < f(\sigma)\sigma < k\sigma^2, \; k = const > 0 \tag{2}$$

We try to solve the task of constructing guaranteed absolute stability condition in given class of Lyapunov functions

$$V(x) = x^T Hx + \beta \int_0^{\sigma(x)} f(\sigma) d\sigma, \; \beta = const > 0. \tag{3}$$

There is well-known sufficient condition of absolute stability, which was constructed utilizing Lyapunov function (3).

Theorem 1 [2,3,6,8]**.** *Let exist positive definite matrix* H *and constants* $\beta > 0$, $\nu > 0$, *such that the matrix*

$$C_1(H,\beta,\nu) = \begin{bmatrix} -A^T H - HA & -\left[Hb + \frac{1}{2}(\beta A^T + I\nu)c \right] \\ -\left[Hb + \frac{1}{2}(\beta A^T + I\nu)c \right]^T & \frac{\nu}{k} - \beta b^T c \end{bmatrix} \qquad (4)$$

is positive definite (I *- unique matrix). Then the Lur'e type control system (1) is absolute stable.*

That is mean, we want to find positive definite $n \times n$ matrix H^0 and constant parameters $\beta^0 \geq 0$, $\nu^0 \geq 0$, for which minimal eigenvalue of matrix $C_1(H^0, \beta^0, \nu^0)$ will be the maximal (and positive).

This optimization task will consider at next set $L = \{(H,\beta,\nu) : H \geq 0, \beta \geq 0, \nu \geq 0\}$.

As we knew, symmetric matrix $C_1(H,\beta,\nu)$ (4) is positive definite if and only if, then $\lambda_{\min}[C_1(H,\beta,\nu)] > 0.$ And construction of guaranteed absolute stability condition of system (1) in functions class (3) we will consider as optimization problem

$$\varphi_1(H,\beta,\nu) \to \min_{(H,\beta,\nu)\in L} \qquad (5)$$

with restriction

$$\lambda_{\min}(H) \geq 0, \ \beta \geq 0, \ \nu \geq 0, \ \varphi_1(H,\beta,\nu) = -\lambda_{\min}[C_1(H,\beta,\nu)]. \qquad (6)$$

It is easy to see, that the set L is a linear space, which is a convex cone. And if optimization task (5), (6) will have the next solution (H^0, β^0, ν^0): $\varphi_1(H^0, \beta^0, \nu^0) < 0,$ then control system (1) will be absolutely stable. If $\varphi_1(H^0, \beta^0, \nu^0) > 0,$ then absolute stability problem for system (1) can not be solved in the class of functions (3).

Denote L_1 is subset of L_0 which consist from next triples (H,β,ν) located inside of unique sphere

$$\lambda^2_{\max}(H) + \beta^2 + \nu^2 \leq 1. \qquad (7)$$

Lemma 1. *The optimization problem (5) - (7) has a solution.*

Proof. Set of positive semidefinite matrices is a convex cone. The set L is a set of triples, which consist from positive semidefinite matrix H and two nonnegative scalar values. The set L is a convex cone, which center consist from next triple – zero matrix and two zero constant values. But the set L_1 is the crossing of unique sphere and closed cone. That is L_1 - compact set. Function $\varphi_1(H,\beta,\nu)$ is continuous function, because it is an eigenvalue of symmetric matrix. Therefore, by Weierstrass extreme value theorem [9], on compact domain it attend own minimal value. That is *QED.*

Solution process of optimization problem can be simplify if function and their domain are convex. In this case there is possibility to formulate necessary and sufficient conditions of optimality.

Lemma 2. *The function* $\varphi_1(H,\beta,\nu)$ *is convex in the domain* L_1.

Prof. Since matrix $C_1(H,\beta,\nu)$ is linear operator by variables (H,β,ν), and the minimal eigenvalue of positive definite matrix is concave function, then for arbitrary triples (H^1,β^1,ν^1), (H^2,β^2,ν^2)will be fulfils

$$\varphi_1(\xi H_1^1+(1-\xi)H_1^2,\xi\beta^1+(1-\xi)\beta^2,\xi\nu^1+(1-\xi)\nu^2)=$$
$$=-\lambda_{\min}\left[C_1\left(\xi H^1+(1-\xi)H^2,\xi\beta^1+(1-\xi)\beta^2,\xi\nu^1+(1-\xi)\nu^2\right)\right]=$$
$$=-\lambda_{\min}\left[\xi C_1\left(H^1,\beta^1,\nu^1\right)+(1-\xi)C_1\left(H^2,\beta^2,\nu^2\right)\right]\le-\xi\lambda_{\min}\left[C_1\left(H^1,\beta^1,\nu^1\right)\right]-$$
$$-(1-\xi)\lambda_{\min}\left[C_1\left(H^2,\beta^2,\nu^2\right)\right]=\xi\varphi_1(H^1,\beta^1,\nu^1)+(1-\xi)\varphi_1\left(H^2,\beta^2,\nu^2\right),$$

that is *QED.*

Therefore task (5) - (7) is the convex optimization problem. The extreme eigenvalues of symmetric positive definite matrices is piecewise continuous differentiated functions. So, we will formulate conditions of solution existence in terms of sub-gradients.

Definition 1. The scalar product of two triples (H^1,β^1,ν^1) and (H^2,β^2,ν^2) is

$$\left\langle(H^1,\beta^1,\nu^1),(H^2,\beta^2,\nu^2)\right\rangle=\sum_{i,j=1}^{n}h_{ij}^1h_{ij}^2+\beta^1\beta^2+\nu^1\nu^2,$$

where

$$H^1=\left\{h_{ij}^1\right\},\ H^2=\left\{h_{ij}^2\right\},\ h_{ij}^1=\overline{1,n},\ h_{ij}^2=\overline{1,n}.$$

Let is denote.

Δ_{ij} - symmetric $(n\times n)-$matrix, which have as $(i,j)-$and $(j,i)-$ elements the constant 0.5, in case $i\ne j$ and 1, in case $i=j$. Θ - $(n\times n)-$ zero matrix. Then the arbitrary matrices $H^1=\left\{h_{ij}^1\right\}$, $H^2=\left\{h_{ij}^2\right\}$, $h_{ij}^1=\overline{1,n}$, $h_{ij}^2=\overline{1,n}$ can be present as a decomposition

$$H^1=\sum_{i,j}^{n}h_{ij}^1\Delta_{ij}\ ,\ H^2=\sum_{i,j}^{n}h_{ij}^2\Delta_{ij}\ .$$

Definition 2. Sub-gradient of convex function $\varphi_1(H,\beta,\nu)$ at internal point $(H^0,\beta^0,\nu^0)\in L_1$ is the triple (E_0,f_0,k_0), for which at arbitrary $(H,\beta,\nu)\in L_1$ will be true

$$\varphi_1(H,\beta,\nu)-\varphi_1(H^0,\beta^0,\nu^0)\ge\left\langle(E_0,f_0,k_0,(H-H^0,\beta-\beta^0,\nu-\nu^0)\right\rangle. \qquad (8)$$

Extreme eigenvalues of symmetric semi positive definite matrices are piecewise continuous differentiated functions. Therefore in point (H^0,β^0,ν^0) may be exists not one vector, but the set of vectors, which satisfied condition (8).

Definition 3. Gradient set $R_\varphi\{E,f,k\}$ of function $\varphi_1(H,\beta,\nu)$ at internal point $(H^0,\beta^0,\nu^0)\in L_1$ will be called the set of triples (E_0,f_0,k_0), which satisfies (8).

Let is calculate sub-gradient of $\varphi_1(H,\beta,\nu) = -\lambda_{\min}[C_1(H,\beta,\nu)]$ at internal point. We will obtain next proposition.

Theorem 2. *The sub-gradient of* $\varphi_1(H,\beta,\nu) = -\lambda_{\min}[C_1(H,\beta,\nu)]$ *at internal point* $(H^0,\beta^0,\nu^0) \in L_1$ *is a triple* (E_0, f_0, k_0), *which consist from matrix* E_0, *and scalars* f_0, k_0, *and have next type*

$$E_0 = \{e_{ij}^0\},\ e_{ij}^0 = -z_0^T C_1(\Delta_{ij},0,0)z_0,\ \ i,j=\overline{1,n},\ \ f_0 = -z_0^T C_1(\Theta,1,0)z_0,\ k_0 = -z_0^T C_1(\Theta,0,1)z_0 \quad (9)$$

Here z_0 *- unique vector at which the quadratic form* $z^T C_1(H^0,\beta^0,\nu^0)z$ *will achieve minimal value (eigenvector, which is correspond to minimal eigenvalue).*

Proof. The minimal eigenvalue of symmetric positive semi definite matrix is equal the minimal value of quadratic form with this matrix, which take at unique sphere. Therefore

$$\varphi_1(H,\beta,\nu) - \varphi_1(H^0,\beta^0,\nu^0) = -\lambda_{\min}[C_1(H,\beta,\nu)] + \lambda_{\min}[C_1(H^0,\beta^0,\nu^0)] =$$

$$= -\min_{|z|=1} z^T C_1(H,\beta,\nu)z + \min_{|z|=1} z^T C_1(H^0,\beta^0,\nu^0)z$$

Let the quadratic form $z^T C_1(H,\beta,\nu)z$ will take minimal value at vector $z = z_1$, and quadratic form $z^T C_1(H^0,\beta^0,\nu^0)z$ - at vector z_0. Then will be

$$\varphi_1(H,\beta,\nu) - \varphi_1(H^0,\beta^0,\nu^0) = -z_1^T C_1(H,\beta,\nu)z_1 + z_0^T C_1(H^0,\beta^0,\nu^0)z_0 =$$

$$= z_0^T\left[C_1(H^0,\beta^0,\nu^0) - C_1(H,\beta,\nu)\right]z_0 + z_0^T C_1(H,\beta,\nu)z_0 - z_1^T C_1(H,\beta,\nu)z_1$$

Since quadratic form $z^T C_1(H,\beta,\nu)z$ take minimal value at vector z_1, then

$$z_0^T C_1(H,\beta,\nu)z_0 - z_1 C_1(H,\beta,\nu)z_1 \geq 0.$$

Hence

$$\varphi_1(H,\beta,\nu) - \varphi_1(H^0,\beta^0,\nu^0) \geq z_0^T\left[C_1(H^0,\beta^0,\nu^0) - C_1(H,\beta,\nu)\right]z_0.$$

According to fact of linearity of operator $C(H,\beta,\nu)$, we will have

$$C_1(H,\beta,\nu) = \sum_{i,j=1}^{n} h_{ij} C_1(\Delta_{ij},0,0) + \beta C_1(\Theta,1,0) + \nu C_1(\Theta,0,1)$$

$$C_1(H^0,\beta^0,\nu^0) = \sum_{i,j=1}^{n} h_{ij}^0 C_1(\Delta_{ij},0,0) + \beta^0 C_1(\Theta,1,0) + \nu^0 C_1(\Theta,0,1).$$

Therefore will have the inequality

$$\varphi_1(H,\beta,\nu) - \varphi_1(H^0,\beta^0,\nu^0) \geq \sum_{i,j=1}^{n}\left(h_{ij}^0 - h_{ij}\right)z_0^T C_1(\Delta_{ij},0,0)z_0 + \left(\beta^0 - \beta\right)z_0^T C_1(\Theta,1,0)z_0 +$$

$$+ (\nu_0 - \nu)z_0^T C_1(\Theta,0,1)z_0.$$

Utilizing (9), we will have

$$\varphi_1(H,\beta,\nu) - \varphi_1(H^0,\beta^0,\nu^0) \geq \sum_{i,j=1}^{n}(h_{ij} - h_{ij}^0)e_{ij}^0 + \left(\beta - \beta^0\right)f_0 + \left(\nu - \nu^0\right)k_0.$$

From definition 1 of scalar product follows

$$\varphi_1(H,\beta,\nu) - \varphi_1(H^0,\beta^0,\nu^0) \geq \left\langle (E_0, f_0, k_0),\ \ \left(H - H^0, \beta - \beta^0, \nu - \nu^0\right)\right\rangle,$$

that *QED*.

Using the obtained expression for sub-gradient and fact of convex set L_1, we will formulate solvability conditions of optimization problem (5)-(7), accordingly to [10].

Theorem 3. *The function $\varphi_1(H,\beta,\nu)$ achieve their minimal value at the point $(H^0,\beta^0,\nu^0)\in L_1$ if and only if, when for arbitrary $(H,\beta,\nu)\in L_1$ will be true next condition*

$$\left\langle (E_0, f_0, k_0),\quad \left(H-H^0,\beta-\beta^0,\nu-\nu_0\right)\right\rangle \geq 0 .$$

Moreover point (H^0,β^0,ν^0) satisfied boundary conditions

$$\sum_{i,j=1}^{n}\left(h_{ij}^0\right)^2+\left(\beta^0\right)^2+\left(\nu^0\right)^2=1.$$

Proof. Necessity. As follows from the above quoted statements, function $\varphi_0(H,\beta,\nu)$ is convex. L_1 – convex cone of triples $H>0$, $\beta>0$, $\nu>0$. Let is true next condition

$$\min_{(H,\beta,\nu)\in L_1}\{\varphi_1(H,\beta,\nu)\}=\varphi_1^*.$$

Let is consider next set

$$M=\{(H,\beta,\nu,\gamma):(H,\beta,\nu)\in L_1,\ -\infty<\gamma<+\infty\}.$$

We will construct next two subset inside

$$M_1=\{(H,\beta,\nu,\gamma):(H,\beta,\nu)\in L_1,\ \gamma\geq\varphi_1(H,\beta,\nu)-\varphi_1^*\},$$
$$M_2=\{(H,\beta,\nu,\gamma):(H,\beta,\nu)\in L_1,\ \gamma<0\}.$$

From the construction, they have no common points. Moreover, subsets M_1 and M_2 are convex, because the function $\varphi_1(H,\beta,\nu)$ is convex. Therefore there is hyperplane

$$\langle (E,f,k),(H,\beta,\nu)\rangle+\varsigma\gamma=0$$

With normal's (E,f,k) and ς, which is separate subsets M_1 and M_2. And for arbitrary points

$$(H,\beta,\nu,\gamma)\in M_1,\quad (H^*,\beta^*,\nu^*,\gamma^*)\in M_2$$

and extreme point $(H^0,\beta^0,\nu^0,0)$ will be true next inequality

$$\langle (E,f,k),(H,\beta,\nu)\rangle+\varsigma\gamma\geq\left\langle (E,f,k),(H^0,\beta^0,\nu^0)\right\rangle\geq\left\langle (E,f,k),\left(H^*,\beta^*,\nu^*\right)\right\rangle+\varsigma\gamma^*. \qquad (10)$$

We will try determine the sign of scalar ς. Let is consider right hand side of inequality (10) under condition $(H^*,\beta^*,\nu^*,\gamma^*)=(H^0,\beta^0,\nu^0,-1)$. We will obtain

$$\left\langle (E,f,k),(H^0,\beta^0,\nu^0)\right\rangle\geq\left\langle (E,f,k),(H^0,\beta^0,\nu^0)\right\rangle-\varsigma.$$

Thereby $\varsigma\geq 0$. Let is show that $\varsigma>0$. By contradiction - $\varsigma=0$. Then putting the left side of the inequality (10)

$$(H,\beta,\nu)=(H^0,\beta^0,\nu^0)+\varepsilon(E,f,k),\ \varepsilon<0,\ \gamma>\varphi_0(H,\beta,k)-\varphi_0^*,$$

we will have

$$\left\langle (E,f,k),(H^0,\beta^0,\nu^0)+\varepsilon(E,f,k)\right\rangle\geq\left\langle (E,f,k),(H^0,\beta^0,\nu^0)\right\rangle,$$

or

$$\varepsilon|(E, f, k)|^2 \geq 0.$$

But because $\varepsilon < 0,$ then this fact possible only if $|(E, f, k)| = 0.$ Thereby our assumption is false, and $\varsigma > 0.$ Let is divide (10) on $\varsigma > 0$. We will obtain

$$\left\langle \frac{1}{\varsigma}(E, f, k), (H, \beta, \nu) \right\rangle + \gamma \geq \left\langle \frac{1}{\varsigma}(E, f, k), \left(H^0, \beta^0, \nu^0\right) \right\rangle \geq \left\langle \frac{1}{\varsigma}(E, f, k)\left(H^*, \beta^*, \nu^*\right) \right\rangle + \gamma^* \qquad (11)$$

It follows, that for arbitrary $\gamma,$ which is satisfy condition

$$\gamma \geq \varphi_1(H, \beta, \nu) - \varphi_1^*,$$

the left-hand side of (11) should satisfy next inequality

$$\gamma \geq \left\langle -\frac{1}{\varsigma}(E, f, k), (H - H^0, \beta - \beta^0, \nu - \nu^0) \right\rangle.$$

Let is

$$\gamma = \varphi_0(H, \beta, \nu) - \varphi_0(H^0, \beta^0, \nu^0),$$

then

$$\varphi_1(H, \beta, \nu) - \varphi_1(H^0, \beta^0, \nu^0) \geq \left\langle -\frac{1}{\varsigma}(E, f, k), (H - H^0, \beta - \beta^0, \nu - \nu^0) \right\rangle.$$

Therefore the triple $\left(-\frac{1}{\varsigma}E, -\frac{1}{\varsigma}f, -\frac{1}{\varsigma}k\right),$ is a sub-gradient (by definition). And we will have that

$$\left(-\frac{1}{\varsigma}E, -\frac{1}{\varsigma}f, -\frac{1}{\varsigma}k\right) = (E_0, f_0, k_0),$$

with elements, which is determined in (9). Putting $\gamma^* = 0,$ at right hand-side of (11), we will have

$$\left\langle (E_0, f_0, k_0), \left(H^* - H^0, \beta^* - \beta^0, \nu^* - \nu^0\right) \right\rangle = \left\langle -\frac{1}{\varsigma}(E, f, k), \left(H^* - H^0, \beta^* - \beta^0, \nu^* - \nu^0\right) \right\rangle.$$

Sufficiency. Let is the conditions of theorem fulfilled and sub-gradient from function $\varphi_1(H, \beta, \nu)$ exist at the point $(H^0, \beta^0, \nu^0) \in L_1$, such that

$$\left\langle (E_0, f_0, k_0)\left(H - H^0, \beta - \beta^0, \nu - \nu^0\right) \right\rangle \geq 0.$$

But then, from the definition of sub-gradient

$$\varphi_1(H, \beta, \nu) - \varphi_1(H^0, \beta^0, \nu^0) \geq 0,$$

or

$$\varphi_1(H, \beta, \nu) \geq \varphi_1(H_0, \beta^0, \nu^0),$$

and point (H^0, β^0, ν^0) is the minimum-point of function $\varphi_1(H, \beta, \nu)$.
From the function $\varphi_1(H, \beta, \nu)$ uniformity by argument (H, β, ν), and from the form of set L_1 follows belonging point (H^0, β^0, ν^0) to the sphere boundary. *Theorem is QED*.

Finally, absolute stability conditions of Lur'e type control system (1) may be formulated as the next assertion.

Theorem 4. *Let is H^0 - positive definite matrix and scalar parameters β^0, ν^0 such that*

$$\left\langle (E_0, f_0, k_0), \left(H - H^0, \beta - \beta^0, \nu - \nu^0\right)\right\rangle \geq 0, \quad \lambda_{\max}^2\left(H^0\right) + \left(\beta^0\right)^2 + \left(\nu^0\right)^2 = 1.$$

Then for absolute stability of the system (1) sufficient that matrix $C_1(H^0, \beta^0, \nu^0)$ (4) will be positive definite. Moreover, if $C_1(H^0, \beta^0, \nu^0)$ is not positive definite, then we can not obtain any theorem about absolute stability for system (1) with utilization of Lyapunov function from apriority given class of function (3). Namely, function

$$V_0(x) = x^T(t) H^0 x(t) + \beta^0 \int_0^{\sigma(x)} f(\sigma) d\sigma$$

is optimal in given class.

CONCLUSION

Well-known absolute stability problem in simple traditional formulation (1), (2) was chosen only with the aim to demonstrate approach and technique. All presented results can be extended on wide class of nonlinear direct and indirect Lur'e type control system with time-delay argument or neutral type (see for example [7,11-14]). Also, results distributed on case of construction of Lyapunov-Krasovskii functionals in some given class. This approach can be applied in case of absolute stability problem, which describes in terms of difference equations.

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PHYSICS TEACHER AND MODELING IN TEACHING PROCESS

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Abstract: *For a detailed analysis of physical phenomena is often necessary to have adequate mathematical apparatus. However, students often do not have adequate mathematical skills. This disadvantage can be replaced by modeling of various phenomena. Work with models and even with the computer can be understood as inter-subject binding, for example, between physics and mathematics, computer science, technology, or other objects. Modeling of phenomena or processes is used in engineering and science; however we can also take advantage of his strengths in education. Computer models, work with graphs and other visual means to help students to understand of phenomena and processes. The realized research deals with the skills of the teacher as a user and/or a creator of the models. The paper presents the current status of the use and the creation of modeling based on different integrated development environment in the Czech Republic.*

Keywords: Dynamic model; modeling; web; ICT education

INTRODUCTION

In today's digital world, information and communication technology (ICT) is extremely integrated into the daily life of the common man and intensely affect it. Society requests schools to offer pupils or students an environment that is going to diversify and encourage further education as a logical consequence of these changes. Teachers must respond to this situation. It is one of the fundamental factors of the educational process. The trend of recent years is more implementation of ICT in the educational process. Therefore, teachers use more modern teaching methods and resources to develop more interactivity of education of pupils and students [1], [2].

The paper deals with a problem of the use and development of computer models in school physics. Working with computer models can be seen as a cross-curricular link between Physics and Mathematics, Engineering or possibly other subjects. Computer models working with graphs and other visualization devices simplify students understanding of the investigated phenomena and processes. In addition the ability of understanding to physics in everyday life is strengthened. Therefore, it is necessary to support the above-mentioned issues in teacher education at universities [7].

1. ICT AND TEACHING (LEARNING)

The fight for student attention leads teachers to search new and more attractive teaching methods. Therefore, it is necessary to look for other possible uses and connections with other school subjects. One of the options can be a bond between Physics and computer science. Physics will supply the required scientific basis; computer science will add the necessary technical equipment and provide a programming background. If we are interested about this cross-curricular links at schools, the solution will be quite simple. Teachers of different subjects will prepare basic tasks. As first, the problem is outlined and explained in Physics

and afterwards in computer science is analysed and possibly transformed into a programming language. In this case the cross-curricular bond is fulfilled [10].
ICT is mainly used as a tool in Physics and other school subjects at elementary and secondary schools. It simplifies work, especially in cases where it is necessary to demonstrate the action or phenomenon that would be under normal circumstances technically unfeasible or impossible for safety reasons.
The most used applications include spreadsheet. This is used for most calculations and creating graphs dependencies of various quantities. As others there are used freely available applications in Physics, animations, Java applets and physlets intended for modeling physical phenomena, remotely controlled laboratories, modeling and simulation programs.
Moreover, pupil or student consciously applies the knowledge, skills and habits directly into his work. He/she immediately sees that he/she is not learning for grades but for life.
In recent years, the development of Physics and ICT demonstrate that the Research and Development (R&D) Physics has used more ICT. In the same way the use of ICT in schools is not limited only for demonstration experiments.

1.1 Dynamic modeling and teaching

Creation of mathematical models of physical processes is one of the classic examples of the use of computer technology in the teaching process. Typical tasks are those processes which are described by differential equations. In dynamic modeling we use a computer to the numerical solution of these differential equations. Thus, we examine the temporal dependence of the system behaviour. Solving differential equations can be carried out by various methods. Even the simplest of those provide results in solving dynamic tasks which describe the sequences of real events very well and enable their graphical modeling [8].
If we look at the problem from an interdisciplinary perspective, we will find that on-line dynamic modeling offers a cross-curricular link. In our case, the output of dynamic modeling is a graph which has a very important role in the images of real life of pupils and students.
From the didactic point of view, a great advantage of the dynamic modeling is the possibility to solve tasks with simple mathematical procedures. The dynamic modeling thus provides a link between the pedagogical, didactic and technical resources in teaching physics [7].

2 COMPUTER MODELING IN PHYSICS

Visualization of phenomena and animations leads to better understanding of basic mathematical operations and physical phenomena for pupils and students. One way is to offer a completed model to pupil or student so he/she can concentrate more on its description or functions and does not have to concentrate on imagining it [8].
The other way is connected with creation of the appropriate model of the physical phenomena. Computer modeling brings new physical knowledge about properties and behaviour of various substances to students. Richard Phillips Feynman was one of the first who used it in his lectures (without the help of a computer, students were making all calculations manually) [3].
Through dynamic modeling in teaching physics, the results of exploration can effectively be presented to pupils or students; even if their mathematical skills are not sufficient. However, they realize part of the whole problem and can at least delimit some quantities. The mathematical model should be capable dynamically respond to a change in the initial conditions or constants characterizing given variables.
For the creation of a dynamic/mathematical model it is necessary to know a fundamental equation for a given action and initial conditions [8]. From the equation we then choose one

variable which value we change and watch the dependence of a system. Then, just for gain advantage, we put the values acquired by calculation into a graph.
The creator of such models must be aware of not only the physical knowledge of the issue being modeled – he/she must also have a relevant knowledge of mathematics and programming language or used integrated development environment (IDE). If both – a graphic and a value output are required, he/she must have a better command of programming than it is usually acquired on completion of programming seminars in the field of study of physics for teachers [8].

3 RESEARCH PROCESS

In the course 2013 we collected data from a research investigation: How teachers of physics at elementary and secondary schools create and use computer models.
The goal of the research is to characterize the teacher as a creator or user of computer models in teaching physics at schools as well as to identify the actual process of creating computer models and introducing them into teaching/learning process.
Research questions and hypotheses were set with respect to determined research goal. In the course of evaluating questionnaires, the frequency of all kinds of answers was ascertained and subsequently hypotheses on a mutual relation between the acquired data were tested. The questionnaires were filled by a total of 166 respondents, of which 96 were men and 70 women (see Tab. 1). The time necessary for completing the questionnaire was about 10 minutes. The participation of respondents was anonymous and voluntary and no reward was provided for filling in a questionnaire.

	Qualification Phys. + Math; Phys. + Computer science	**Other Qualification or without qualification**	**Σ**
never	10	7	17
1-2 times per half of the year	22	14	36
1-2 times per month	59	24	83
1 time per week	24	6	30
each lesson	0	0	0
Σ	115	51	166

Tab. 1. Absolute numbers of respondents according to the environment in which they worked at the university, which used to create models in their practice
Source: own

We present two hypothesis of the total number of six hypotheses in this paper. These are focused on IDE of computer models. More results will gradually be published:

The hypothesis H1 was formulated as null (H10) and alternative (H1A) hypothesis.

H10: *Qualified teachers of Physics + Math or Physics + Computer science do not use computer models in school physics more often than teachers qualified to teach Physics + other school subjects.*
H1A: *Qualified teachers of Physics + Math or Physics + Computer science use models in school physics more often than teachers qualified to teach physics + other school subjects.*

Commentary on the hypothesis H1:
We assume that teachers qualified to teach Physics + Math or Physics + Computer science use modeling more often because it is closer to their qualifications. They should not have a significant problem in understanding the internal structure of the model if they had an access

to it. Furthermore, we assume that this group will also create models, but this is beyond the scope of this hypothesis.

The hypothesis H2 was formulated as null (H20) and alternative (H2A) hypothesis.

H20: *Teachers who create computer models do not use just the development environment (IDE) which they learned to work with at the university.*

H2A: *Teachers who create computer models just use the development environment (IDE) which they learned to work with at the university.*

Commentary on the hypothesis H2:
We suppose that teachers choose integrated development environment known to them intentionally. Working in such IDE is much easier for them than working in the new one. At the same time, we assume – and our assumptions are confirmed in practice – that they have some models they created at the time of their studies at university. Another reason for this hypothesis is a uniform character of models presented to pupils that goes hand in hand with another aspect, namely the requirement for the minimum user knowledge of more integrated development environment.

4 HYPOTHESES DISCUSSION

Hypothesis H1
On the basis of the appropriate questions and answers of respondents (see Tab. 2) it was calculated χ-square test [4] for the hypothesis H1.

	IDE learned at university	**IDE not learned at university**	**Σ**
IDE used in practice	29	12	41
any IDE used in practice	30	95	125
Σ	59	107	166

Tab. 2. Two by two table with absolute numbers of respondents according to the IDE use and creation of models at university and in teaching practice
Source: own

We calculated the critical value of a test criterion for the level of significance ($p \leq 0.01$) and for the number of degrees of freedom ($f = 4$) to be $\chi^2_{0.01}(4) = 13.28$. As the calculated value ($\chi^2_{H1} = 3.75$) is significantly lower than the critical value, the null hypothesis was confirmed. So it was not proved that the qualified teachers of Physics + Math or Physics + Computer science use computer models in teaching Physics more often than teachers qualified to teach Physics + other school subjects.
Qualification of teachers in the research sample does not play a statistically significant role in the use of computer models in lessons. Expected results were totally opposite. In many cases, the results were influenced by teachers qualified to teach Physics + technical education or similar. The result can be explained by the fact that these teachers have been working with different models more often than the others since their studies at the university. Likewise, we assume that they may have a different imagination.
The availability and properties of the existing and new IDE may play a role too (for example new possibilities of analysing the phenomena examined, new possibilities as far as modeling outputs are concerned, etc.).

Hypothesis H2
The obtained answers of respondents were transferred to a two by two table (Tab. 2). For the results in Table 2 it was calculated χ-square test and Yule's Q association coefficient [4].
We calculated the critical value of a test criterion for the level of significance ($p \leq 0.01$) and for the number of degrees of freedom ($f = 1$) to be $\chi^2_{0.01}(1) = 6.63$. As the calculated value ($\chi^2_{H2} = 29.43$) is significantly higher than the critical value, we reject the null hypothesis H20. It was proved that teachers who create computer models preferably use the development environment (IDE) which they learned to work with during their university studies.
As it follows from Table 2, there is a statistically significant dependence between the examined phenomena. We calculate the value of the tightness of a relationship between variables in a two by two table (r_Φ). The coefficient's range is ($-1 \leq r_\Phi \leq 1$) and it applies that the higher the coefficient, the higher the degree of dependence. For our case we obtain the value ($r_\Phi = 0.42$) which corresponds to a medium relationship between the evaluated phenomena.
We evaluated Yule's Q association coefficient for a given table. The coefficient's range is ($-1 \leq Q \leq 1$) and provides information about the one-sidedness of the examined phenomenon. After a calculation we acquire the value ($Q = 0.77$) which corresponds to a tight relationship between the evaluated variables.
The above results (χ-square test, the value of the tightness of a relationship and Yule's Q association) show that the confirmation of H2A was correct.

Due to the results of the research it is necessary to propose to the teacher the integrated development environment suitable for creation of models which is on-line, free of charge and without download on the user's computer. Moreover, with regard to trends in education, it is advisable to follow the path of creating models directly on the website. So, we suggest PHP platform as a suitable IDE [10], [11]. We introduced a concept for the development of computer models using PHP (Hypertext Preprocessor) [6] to facilitate greater extension of modeling in our schools.

CONCLUSION

The image of education in public has been changed as a result of constant confrontation of traditional teaching with digital and ICT technologies. The image is affected by excessive use of ICT outside the school and its strict rejection in the classroom. We can conclude from our survey that: "ICT is implemented in the present school but the level of integration is not as significant as imagined by parents, pupils or students themselves." We imagine dynamic modeling as an ideal way to expand and improve the use of ICT at schools. It is understood as a complementary component of education at all school levels in our concept [10], [11].

Our findings show that teaching via modeling stimulates the pupils' interest in the particular topics and increases the effectiveness of teaching at the same time. Moreover, this approach may replace the insufficient mathematical apparatus necessary for solving the issues in question. The modeling will reinforce the student's ability to identify the physical grounds of processes and phenomena they know from everyday life regardless of the student's level of analytical and logical thinking [11].

The physics teacher as a creator of such models must be aware of not only the physical knowledge of the issue being modeled – he/she must also have a relevant knowledge of mathematics and programming language or used integrated development environment.

From the research survey we can state that the subject qualification of teachers does not play a significant role in frequency of model use in teaching. It was proved that teachers who create computer models preferably use the development environment (IDE) which they learned to work with during their university studies. There is a large group of teachers that does not use nor create models in teaching physics. One possible explanation is the lack of appropriate IDE.

So, we suggest PHP platform as a suitable IDE [10], [11]. We introduced a modeling concept based on PHP [6] to facilitate greater extension of modeling in our schools. Chosen solution is simple for the user. It gives him the opportunity to work with models on the computer, smart phone or tablet. Motivation models suitable for teaching are placed on a publicly available web page http://www.ped.muni.cz/modely [9].

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